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Option Pricing with Lévy Processes

Jump models for European-style options

Rui Monteiro

Dissertation

Master of Science in Financial Mathematics

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Supervisor: João Pedro Nunes

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Abstract

The goal of this dissertation is to explain how the pricing of European-style options under Lévy processes, namely jump and jump diffusion processes, can be performed and the mathematics associated with it. For this purpose, three models are exposed: Merton, Kou and Variance Gamma, each with different valuation approaches. A Monte Carlo path simulation is also explained. Finally, calibration of the models to real data takes place.

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Chapter 1

Introduction

1.1 Motivation: Why Large Oscilations

A common problem within standard financial theory is that one admits that the price of a financial asset follows a stochastic process based on a Brownian motion, in which variations of great amplitude are not likely to happen. This assumption dates back from the origins of modern financial theory, pioneered by [4]. This assumption was also the base of the first option pricing model, the well known Black-Scholes model — see [7]. However, empirical evidence shows that extreme variations in prices do occur and that the tails of the distributions of the logarithms of the returns on the assets are heavier than usually assumed — see, for instance, [22]. This has serious consequences in the evaluation of financial derivatives, namely options, since their prices derive from the underlying asset's behavior. Path continuity is also a common assumption. It is, however, inconsistent with the existence these variations — also called jumps. These are illustrated, using real data, in Appendix A.1. One way to try and get around the problem is to consider Lévy processes, namely processes with jumps, in which non marginal variations are more likely to happen as a consequence of fat-tailed distribution based processes, being, therefore, much more realistic.

1.2 Objective and Methodology

This thesis aims to describe pricing models for European-style options, assuming that the underlying asset follows a Lévy process — jump-diffusion and pure jump — hoping that this way, one can better explain the options' market. The thesis is structured as follows. First, an introduction to the mathematical tools associated with Lévy processes. The following chapter is dedicated to two jump-diffusion models — the Merton and the Kou models — where a common introduction takes place, since both models are similar in their essence. However, the methodologies used for pricing are different. Some other fundamental mathematical methods are explained. Then, pure jump processes are introduced and one of the applications, namely the Variance Gamma model, is exposed, with the particularity that the new pricing approach is totaly different from the ones used in the jump-diffusion models. A Monte Carlo path simulation is also explained and calibration to real data also takes place. Finally, I draw some conclusions and make a suggestion of further reading.

This thesis is, thus, a very modern, compact and clear exposure of the theory and applications of Lévy processes in financial modeling, and can also be viewed as an extension to the models studied during this master's course.

I hope the reader may enjoy this work.

Chapter 2

Mathematical Tools

2.1 Stochastic Process

First of all, it is important to recall what a stochastic process is. A stochastic process is simply a collection of random variables referenced to an ordered set. This way, it can be used to construct models in which some given system might evolve in a non deterministic way, since a whole set of paths are possible. In finance, such processes are of great importance, since one is willing to model the paths of an asset's price, say stock, which are not deterministic at all, for at each instant in time it evolves randomly. Formally a stochastic process can be defined as:

Definition 1. A stochastic process is a collection of random variables S_t referenced to an ordered set $\{t : \ t \in [0, T]\}$ and defined in a probability space $(\Omega, \mathscr{F}, \mathbb{P})$, where Ω is the set of all possible outcomes, $\mathscr{F}$ is a subset of Ω and $\mathbb{P}$ is a function which associates a number — probability — to an event.

2.2 Lévy Process

There are several types of stochastic processes. The one to be dealt with in this work is called a Lévy process. It has some particular aspects that define it. It is a sequence of random variables which are uncorrelated, e.g., the evolution to the next step is not influenced by its past, and are also stationary, which means that the increments that take the process from one point to another within the ordered set — say time — follow the same distribution. It is quite simple, and mathematically can be defined as follows:

Definition 2. A Lévy process is a stochastic process $L = \{L_t : t \geq 0\}$ defined on a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ which verifies the following properties:

1. L has $\mathbb{P}$-almost surely right continuous paths with left limits;
2. $L_0 = 0$;
3. for $0 \leq s \leq t$, $L_t - L_s$ has the same distribution law of L_{t-s} (stationarity);
4. for $0 \leq s \leq t$, $L_t - L_s$ is independent of $L_u : u \leq s$.

For those already familiarized with Wiener normal processes, clearly the only difference is that in the Wiener case it is assumed that the increments follow a normal distribution. Therefore, Wiener processes are a subclass of Lévy processes and generate pure diffusion, since great magnitude variations are not likely.

2.3 Poisson Process

Since the new component is jump like, it is important is to introduce pure jump processes. A jump process is a sequence of random variables that form a purely discontinuous path. To illustrate it, let us define a Poisson process.

Definition 3. A Poisson process is a sequence of random variables with jump size 1, where the occurrence of such jumps is distributed in time as

$$\mathbb{P}(N(t+dt) - N(t) = k) = \frac{e^{-\lambda dt}(\lambda dt)^k}{k!} \quad , \quad k \in \mathbb{N}_0^+$$

and the increments are independent and stationary. λ is called the intensity and k is the number of events between t and $t + dt$.

2.4 Compound Poisson Process

In insurance, for example, Poisson processes are used to model the arrival of accidents for the company to cover, which arrive at a rate λ. The value to be paid by the company is another random variable, making a so called compound Poisson process. It can be formally defined as:

Definition 4. A compound Poisson process is a sequence of random variables where the jump size follows a certain distribution, and the occurrence of such a jump is distributed in time as a standard Poisson process.

2.5 Lévy-Khintchine Representation

A fundamental theorem concerning Lévy processes is the so called Lévy-Khintchine representation:

Theorem 1. Lévy-Khintchine: Let L_t be a Lévy process. Then its characteristic function is of the form

$$\phi_{L_t} = \int_{\mathbb{R}} e^{i\theta L_t} \nu(dx) = e^{t\Psi(\theta)} \quad \forall \theta \in \mathbb{R} \tag{2.1}$$

iff $\exists$ a triplet (a, σ^2, ν), $a \in \mathbb{R}$, $\sigma \geq 0$, ν a measure concentrated on $\mathbb{R}\backslash\{0\}$ that satisfies $\int_{\mathbb{R}} (1 \wedge x^2)\nu(dx) \leq \infty$ s.t.

$$\Psi(\theta) = ia\theta - \frac{1}{2}\sigma^2\theta^2 + \int_{\mathbb{R}} (e^{i\theta x} - 1 - i\theta x \mathbb{1}_{|x|\leq 1})\nu(dx) \quad \forall \theta \in \mathbb{R} \tag{2.2}$$

where Ψ is called the characteristic exponent.

Proof. See [10]. □

This theorem is the core of the theory of Lévy processes, and deserves much attention. It states that every Lévy process has a characteristic function of the form of equation (2.1). Thus, they can be parameterized using the triplet (a, σ^2, ν), where a is the drift,

σ^2 is the variance of a Brownian motion and $\nu(dx)$ is the so called Lévy measure or jump measure. As for the drift and Brownian motion components, one assumes that the reader is already familiar with them. The new component is the Lévy measure, $\nu(dx)$, which is associated with the jumps beyond “normal” that make the process path discontinuous.

2.6 Finite and Infinite Activity

The rate at which such discontinuities arrive is given by the integral of the Lévy density, $\lambda = \int_{-\infty}^{+\infty} \nu(dx)$. In the particular case of compound Poisson processes, $\nu(dx) = \lambda f(x)dx$, where $f(x)$ is the density of the distribution of jumps. When this integral is finite the process is said to have finite activity. If the integral is infinite, then it is said to have infinite activity, which implies that the process can have infinite jumps in a finite time interval.

So, a pure jump process has a triplet of the form $(a, 0, \nu)$, since there is no diffusion component. As for a pure diffusion process, the triplet is of the form $(a, \sigma^2, 0)$ since no jump component exists, whereas in a jump-diffusion process the triplet takes the form (a, σ^2, ν) for it is a mixture of jump and diffusion components.

It is also important to point out that a jump-diffusion has a jump component with finite rate λ, whereas a pure jump process has, in financial modeling, infinite activity.

2.7 Gamma Process

Another important process in financial modeling is the Gamma process. It is a purely discontinuous process with infinite activity. Since the Lévy-Khintchine representation has already been explained, this process will be stated in terms of its Lévy measure. It can be defined as follows.

Definition 5. The Gamma process — see [20] — $\gamma_t(\mu, \delta)$ with mean rate δ and variance rate ν is the increasing process of independent Gamma increments which has the following Lévy measure:

$$\nu(dx) = \frac{\mu^2 e^{-\frac{\mu}{\delta}x}}{\delta x} \mathbb{1}_{x>0} dx$$

See also [3]. This measure has infinite integral, so the process has an infinite arrival rate of jumps, most of which are small, as one can see from the concentration of the Lévy measure at the origin. So, at every instant in time an increment is added, which follows a Gamma distribution. This process will be used in one of the option pricing models presented in this thesis.

2.8 Momenta of Stochastic Processes

Also recall that once one has the characteristic function, the n^{th} moment of the process can be found through

Theorem 2.

$$\mathbb{E}[L_t^{(n)}] = i^{-n} \left[\frac{d^n}{d\theta^n} \phi_{L_t}(\theta) \right]_{\theta=0} \tag{2.3}$$

Proof. See [1]. □

2.9 Exponential Lévy Process

Another important process to point out is the Lévy analogous to the well known geometric Brownian motion, since in finance one often works with the log returns on the asset. It is called exponential Lévy process and can be defined as follows.

Definition 6. Following [10], an exponential Lévy process is a stochastic process that can be written in the form:

$$S_t = S_0 e^{L_t}$$

where L_t is a Lévy process.

2.10 Lévy-Itô Lemma

Since we will be using processes with a jump component it is important to define a way of differentiating functions with stochastic jump and diffusion arguments. Traditionally this is done through the Itô lemma. However, it only admits Wiener like arguments. So, an extension of this lemma to processes with jumps needs to be stated.

Theorem 3. Lévy-Itô lemma: For a process of the type $L_t = L_0 + \int_0^t b_s ds + \int_0^t \sigma_s dW_s + \sum_{i=1}^{N_t} \Delta X_i$ the Itô formula is:

$$\begin{aligned} df(L_t, t) = \frac{\partial f(L_t, t)}{\partial t} dt + b_t \frac{\partial f(L_t, t)}{\partial x} dt + \frac{\sigma_t^2}{2} \frac{\partial^2 f(L_t, t)}{\partial x^2} dt + \sigma_t \frac{\partial f(L_t, t)}{\partial x} dW_t \\ + [f(X_{t^-} + \Delta X_t) - f(X_{t^-})] \end{aligned} \tag{2.4}$$

Proof. See [10]. □

Notice that the only difference to the traditional Itô lemma is the last term, which accounts for the occurrence of jumps, where t^- is the left limit of the function at a given point in time, e.g., the value that the function takes just before the occurrence of a jump. This lemma is valid for finite activity processes only. For infinite activity see, for instance, [23].

I would also like to point out that in this work one will only deal with finite variance processes, since this is an essential requirement in derivative pricing. If this was not the case, then asset's price would be likely to jump with arbitrary magnitude, which would not be realistic.

Now that one is familiarized with Lévy processes, we will proceed to the pricing models.

Chapter 3

Jump-Diffusions

3.1 The Idea

The idea of a jump-diffusion process — see [25] — is that the diffusion part takes into account the normal fluctuations in the risky asset's price caused by, for example, a temporary imbalance between supply and demand, changes in capitalization rates, changes in the economic outlook or other information that causes marginal changes in price. As for the non-marginal variations, it is expected that the information that would cause them come in discrete points in time, and that is why a jump component is added to the traditional diffusion process.

3.1.1 Incorporation of Jumps

The goal is to start from a diffusion model, namely the Black and Scholes model — see [7] — and add up a jump component, which is modeled as a compound Poisson process.

One begins by defining the probability of a certain number of jumps N occurring in the time interval dt, which we call dN_t:

$$\mathbb{P}(N = 1 \ in \ dt) = \mathbb{P}(dN_t = 1) \approx \lambda dt$$
$$\mathbb{P}(dN_t \geq 2) \approx 0$$
$$\mathbb{P}(dN_t = 0) \approx 1 - \lambda dt$$

where λ is the jump intensity. Notice that the arrival of jumps is approximated by a Bernoulli variable, which comes from a series expansion of the Poisson distribution that models the arrival of jumps.

Suppose that in dt, S_t jumps to $V_t S_t$, where V_t is the absolute jump size.

The relative jump size is then:

$$\frac{dS_t}{S_t} = \frac{V_t S_t - S_t}{S_t} = V_t - 1$$

3.1.2 Equation of Dynamics

Thus, in order to incorporate the jump component into the model, one writes the stochastic differential equation that defines the dynamics as:

$$\frac{dS_t}{S_t} = (\alpha - \lambda\zeta)dt + \sigma W_t + (V_t - 1)dN_t \tag{3.1}$$

where W_t is a standard Brownian motion, N_t is a Poisson process with intensity λ, and it is assumed that S_t, N_t and V_t are independent. α is the instantaneous expected return, σ is the volatility associated with the Brownian motion and

$$\zeta = \mathbb{E}[V_t - 1]$$

Notice that

$$\mathbb{E}[\frac{dS_t}{St}] = \alpha dt$$

Thus, the term $-\lambda\zeta$ is added in the drift of equation (3.1) in order to compensate the Poisson process, so that the log returns on the asset will have an expectation of

αdt. Thus, the process of the discounted prices becomes a martingale.

3.1.3 Solution via Lévy-Itô Lemma

Theorem 4. The solution for the stochastic differential equation

$$dS_t = S_t(\alpha - \lambda\zeta)dt + S_t\sigma W_t + S_t(V_t - 1)dN_t$$

is

$$S_t = S_0 e^{(\alpha - \frac{\sigma^2}{2} - \lambda\zeta)t + \sigma W_t + \sum_{k=1}^{N_t} Y_k}$$

Proof. Applying the Itô formula (2.4) for jump-diffusion processes from Theorem 3 to the log returns $f(S_t) = ln(S_t)$, then

$$\begin{aligned}
df &= \frac{\partial f}{\partial t} + (\alpha - \lambda\zeta)S_t\frac{\partial f}{\partial S_t}dt + \frac{\sigma^2 S_t^2}{2}\frac{\partial^2 f}{\partial S_t^2}dt + \sigma S_t\frac{\partial f}{\partial S_t}dW_t \\
&\quad + (f(V_t S_t) - f(S_t)) \\
d(ln(S_t)) &= (\alpha - \lambda\zeta)S_t\frac{1}{S_t}dt - \frac{1}{S_t^2}\frac{\sigma^2 S_t^2}{2}dt + \sigma S_t\frac{1}{S_t}dW_t + ln(V_t) + ln(S_t) - ln(S_t) \\
d(ln(S_t)) &= (\alpha - \lambda\zeta)dt - \frac{\sigma^2}{2}dt + \sigma dW_t + ln(V_t) \\
ln(S_t) &= ln(S_0) + (\alpha - \frac{\sigma^2}{2} - \lambda\zeta)t + \sigma W_t + \sum_{k=1}^{N_t} ln(V_k) \\
S_t &= S_0 e^{(\alpha - \frac{\sigma^2}{2} - \lambda\zeta)t + \sigma W_t}\prod_{k=1}^{N_t} V_k \\
S_t &= S_0 e^{(\alpha - \frac{\sigma^2}{2} - \lambda\zeta)t + \sigma W_t + \sum_{k=1}^{N_t} Y_k}
\end{aligned}$$

where $Y_k = ln(V_k)$.

□

Now, one ends up with an exponential Lévy process $S_t = S_0 e^{L_t}$ where L_t is of the form

$$L_t = \underbrace{(\alpha - \frac{\sigma^2}{2} - \lambda\zeta)t}_{drift} + \underbrace{\sigma W_t}_{Brownian\ Motion} + \underbrace{\sum_{k=1}^{N_t} Y_k}_{compound\ Poisson\ process}$$

and

$$L_t = ln\left(\frac{S_t}{S_0}\right)$$

3.1.4 Risk Neutral Dynamics

For the purpose of risk neutral valuation, it is required that the asset grows at the risk-free rate, so that the discounted prices become a martingale under the risk neutral measure. So, the logical thing to do is to refer to α as the risk-free rate, and taking the other parameters as risk neutral too. Jump-diffusion models are incomplete market models, since there is more than one equivalent martingale measure $\mathbb{Q} \sim \mathbb{P}$ under which the process of the discounted asset price becomes a martingale. In terms of the hedging portfolio, this is equivalent to saying that there is no way of building a completely risk-free hedging portfolio.

[25] gets an equivalent measure, $\mathbb{Q} \sim \mathbb{P}$, by changing the drift of the Brownian motion and keeping the rest unchanged. Then

$$S_t = S_0 e^{(r-q-\frac{\sigma^2}{2}-\lambda\zeta)t+\sigma W_t^{\mathbb{Q}}+\sum_{k=1}^{N_t} Y_k} \tag{3.2}$$

under $\mathbb{Q}$.

Notice that the only thing that happens is that α becomes $r - q$, the risk-free rate (minus the continuous dividend yield). [25] suggests that dN_t has no correlation with the market as a whole, e.g., the risk associated with jumps is non-systematic. Therefore, it can be diversified away. This is the way [25] gets around the incompleteness problem.

3.2 The Merton Model

This section exposes the first option pricing application of jump-diffusion processes, proposed by [25]. I shall follow [24].

3.2.1 Dynamics

Under the Merton model it is assumed that the dynamics are driven by a process generated by an equation like (3.2), with jumps log-normally distributed $Y_k = ln(V_k) \sim \mathscr{N}(\mu, \delta^2)$, where $\mathscr{N}(\mu, \delta^2)$ denotes a Gaussian distribution with mean μ and variance δ^2. Thus

$$\zeta = e^{\mu + \frac{\delta^2}{2}} - 1$$

3.2.2 Characteristic Function, Momenta and Activity

Notice that the process can be characterized by the triplet $(b, \sigma^2, \nu(dx))$, where $b = r - q - \frac{\sigma^2}{2} - \lambda\zeta$ and $\nu(dx) = \lambda f(dx) = \lambda \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(dx-\mu)^2}{2\delta^2}}$. Thus, by using Theorem 1, the characteristic function of the process is

$$\phi_T = e^{ib\theta T - \frac{\sigma^2\theta^2 T}{2} + \lambda T\left(e^{i\mu\theta - \frac{\delta^2\theta^2}{2}} - 1\right)}$$

From its Lévy density, one can check that this is indeed a finite activity process, since

$$\int_{-\infty}^{+\infty} \nu(dx) = \lambda$$

And, by differentiating the characteristic function using Theorem 2, the first two moments can be found to be

$$\mathbb{E}(L_t) = t\,(b + \lambda\delta)$$

$$Var(L_t) = t\left(\sigma^2 + \lambda\mu^2 + \lambda\delta^2\right)$$

As we can see, both the mean and variance are finite.

3.2.3 Option Pricing

The value of a contingent claim — say an European-style option — $H(S_T)$ at time t is the discounted value of its expectation under the risk neutral measure:

$$H_t^M(S_t) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[H(S_T)|\mathscr{F}_t]$$

where the superscript M stands for Merton. Then,

$$\begin{aligned} H_t^M(S_t) &= e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[H(S_t e^{(r-q-\frac{\sigma^2}{2}-\lambda\zeta)(T-t)+\sigma W_{T-t}^{\mathbb{Q}}+\sum_{k=1}^{N_{T-t}} Y_k})|\mathscr{F}_t] \\ &= e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[H(S_t e^{(r-q-\frac{\sigma^2}{2}-\lambda\zeta)(T-t)+\sigma W_{T-t}^{\mathbb{Q}}+\sum_{k=1}^{N_{T-t}} Y_k})] \end{aligned}$$

For simplicity let us use $\tau = T-t$ and define the counter as $N_{T-t} = 0,1,2,3... \equiv i$ so that we use index i to denote the number of jumps; note also that the compound Poisson process follows the law $\sum_{k=1}^{N_t} Y_k \sim \mathscr{N}(i\mu, i\delta^2)$. And, by conditioning on the number of jumps i one obtains:

$$H_t^M(S_t) = e^{-r\tau}\sum_{i\geq 0}\mathbb{Q}(N_\tau = i)\mathbb{E}^{\mathbb{Q}}[H(S_t e^{(r-q-\frac{\sigma^2}{2}-\lambda\zeta)\tau+\sigma W_\tau^{\mathbb{Q}}+\sum_{k=1}^{N_\tau} Y_k})]$$

Notice that $\mathbb{Q}(N_\tau = i)$ is the probability of occurring i jumps between t and T, which follows a Poisson distribution. Then $\mathbb{Q}(i) = \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!}$. Thus

$$H_t^M(S_t) = e^{-r(T-t)}\sum_{i\geq 0}\frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!}\mathbb{E}^{\mathbb{Q}}[H(S_t e^{\{(r-q-\frac{\sigma^2}{2})-\lambda(e^{\mu+\frac{\delta^2}{2}}-1)\}\tau+\sigma W_\tau^{\mathbb{Q}}+\sum_{k=1}^{N_t} Y_k})]$$

The term in the exponential follows

$$\begin{aligned} &\{(r-q-\frac{\sigma^2}{2})-\lambda(e^{\mu+\frac{\delta^2}{2}}-1)\}\tau+\sigma W_\tau^{\mathbb{Q}}+\sum_{k=1}^{N_t} Y_k \\ &\sim \mathscr{N}(\{r-q-\frac{\sigma^2}{2})-\lambda(e^{\mu+\frac{\delta^2}{2}}-1\}\tau+i\mu\ ;\ \sigma^2\tau+i\delta^2) \end{aligned}$$

and it can be rewritten so that its distribution stays the same

$$\begin{aligned} &\{(r-q-\frac{\sigma^2}{2})-\lambda(e^{\mu+\frac{\delta^2}{2}}-1)\}\tau+i\mu+\sqrt{\frac{\sigma^2\tau+i\delta^2}{\tau}}W_\tau^{\mathbb{Q}} \\ &\sim\ \mathscr{N}(\{r-q-\frac{\sigma^2}{2})-\lambda(e^{\mu+\frac{\delta^2}{2}}-1\}\tau+i\mu\ ;\ \sigma^2\tau+i\delta^2) \end{aligned}$$

so it comes

$$H_t^M(S_t) = e^{-r(T-t)} \sum_{i\geq 0} \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!} \mathbb{E}^{\mathbb{Q}}[H(S_t e^{\{(r-q-\frac{\sigma^2}{2})-\lambda(e^{\mu+\frac{\delta^2}{2}}-1)\}\tau+i\mu+\sqrt{\frac{\sigma^2\tau+i\delta^2}{\tau}}W_t^{\mathbb{Q}}})]$$

and by adding and subtracting $\frac{i\delta^2}{2\tau}$ in the exponential one gets

$$\begin{aligned} H_t^M(S_t) =& e^{-r(T-t)} \sum_{i\geq 0} \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!} \\ & \times \mathbb{E}^{\mathbb{Q}}[H(S_t e^{\{(r-q-\frac{1}{2}\left(\sigma^2+\frac{i\delta^2}{\tau}\right)+\frac{i\delta^2}{2\tau})-\lambda(e^{\mu+\frac{i\delta^2}{2}}-1)\}\tau+i\mu+\sqrt{\sigma^2+\frac{i\delta^2}{\tau}}W_t^{\mathbb{Q}}})] \end{aligned}$$

Now, setting $\sigma_i^2 = \sigma^2 + \frac{i\delta^2}{\tau}$ and rearranging it comes:

$$H_t^M(S_t) = e^{-r(T-t)} \sum_{i\geq 0} \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!} \mathbb{E}^{\mathbb{Q}}[H(S_t e^{i\mu+\frac{i\delta^2}{2}-\lambda(e^{\mu+\frac{\delta^2}{2}}-1)\tau} e^{(r-q-\frac{\sigma_i^2}{2})\tau} + \sigma_i W_\tau^{\mathbb{Q}})]$$

Notice that the objective is to get the evaluation formula in terms of the Black-Scholes (BS) formula:

$$H^{BS}(\tau, S_t, \sigma, r-q) := e^{-r\tau}\mathbb{E}^{\mathbb{Q}}[H(S_t e^{(r-q-\frac{\sigma^2}{2})\tau} + \sigma W_\tau^{\mathbb{Q}})] \tag{3.3}$$

3.2.4 Pricing Formula

Thus, Merton's pricing formula can be viewed as a weighted average of BS price conditioned to the number of jumps:

$$H_t^M(S_t) = \sum_{i\geq 0} \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!} H^{BS}(\tau\ ,\ S_i \equiv S_t e^{i\mu+\frac{i\delta^2}{2}-\lambda(e^{\mu+\frac{\delta^2}{2}}-1)\tau}\ ;\ \sigma_i \equiv \sqrt{\sigma^2+\frac{i\delta^2}{\tau}}\ ;\ r-q)$$

Alternatively,

$$
\begin{aligned}
H_t^M(S_t) &= e^{-r\tau}\sum_{i\geq 0}\frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!}\mathbb{E}^{\mathbb{Q}}[H(S_t e^{\{r-q-\lambda(e^{\mu+\frac{\delta^2}{2}}-1)\}\tau+\frac{i\mu+\frac{i\delta^2}{2}}{\tau}-\frac{\sigma_i^2}{2}+\sigma_i W_\tau^{\mathbb{Q}}})] \\
&= \sum_{i\geq 0}\frac{e^{-\widetilde{\lambda}\tau}(\widetilde{\lambda}\tau)^i}{i!}H^{BS}(\tau\ ;\ S_t\ ;\ \sigma_i\ ;\ (r-q)_i)
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{\lambda} &= \lambda(1+\zeta) = \lambda(e^{\mu+\frac{\delta^2}{2}}) \\
\sigma_i &= \sqrt{\sigma^2+\frac{i\delta^2}{2}} \\
(r-q)_i &= r-q-\lambda(e^{\mu+\frac{\delta^2}{2}}-1)+\frac{i\mu+\frac{i\delta^2}{2}}{\tau}
\end{aligned}
$$

Notice that a martingale approach has been performed. One could as well get this very same formula by creating a hedging portfolio, as originally done by [25].

3.3 The Kou Model

In the Kou model, the idea is pretty much the same as in Merton's. It incorporates a geometric drifted Brownian motion and a jump part. However, the latter follows a double exponential distribution, instead of Merton's normal law.
Some advantages of this model are that it captures some important features of the dynamics of financial assets in the real market, namely the asymmetric leptokurtosis and the volatility smile, and yet provides closed form solutions for the most important types of options.

3.3.1 Dynamics

Following [17], under the risk neutral measure the asset's price is modeled by the following stochastic differential equation:

$$\frac{dS_t}{S_{t^-}} = (r - q - \lambda\zeta)dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t}(V_i - 1)\right) \tag{3.4}$$

being W_t a Brownian motion, N_t a Poisson process with intensity λ and V_i a sequence of independent identically distributed non negative random variables s.t. $Y_i = ln(V_i)$ follows an asymmetric double exponential distribution whose density is given by

$$f_Y(x) = p\eta_1 e^{-\eta_1 x}\mathbb{1}_{\{x\geq 0\}} + d\eta_2 e^{\eta_2 x}\mathbb{1}_{\{x<0\}}$$

with $\eta_1 \geq 1$ (so S_t has finite expected value), $\eta_2 \geq 0$, $p, d \geq 0$ and $p + d = 1$. Therefore, the compensator ζ is

$$\zeta = \mathbb{E}^{\mathbb{Q}}[e^Y - 1] = \frac{p\eta_1}{\eta_1 - 1} + \frac{p\eta_2}{\eta_2 + 1} - 1$$

Notice that equation (3.4) is the same as (3.1), with a slightly different notation for the jump component. Thus, as we saw previously, by using Theorem 4 the solution is:

$$S_t = S_0 e^{(r-q-\frac{\sigma^2}{2}-\lambda\zeta)t+\sigma W_t}\prod_{i=1}^{N_t} V_i$$

Then, the logarithm of the return on the asset $L_t \equiv ln(\frac{S_t}{S_0})$ follows

$$L_t = \left(r - q - \frac{\sigma^2}{2} - \lambda\zeta\right) t + \sigma W_t + \sum_{i=1}^{N_t} Y_i, \quad L_0 = 0 \tag{3.5}$$

3.3.2 Characteristic Function, Momenta and Activity

The process can be characterized by the triplet $(b, \sigma^2, \nu(dx))$, where $b = r - q - \frac{\sigma^2}{2} - \lambda\zeta$ and $\nu(dx) = \lambda f(dx) = \lambda\{p\eta_1 e^{-\eta_1 dx}\mathbb{1}_{\{x\geq 0\}} + d\eta_2 e^{\eta_2 dx}\mathbb{1}_{\{x<0\}}\}$. Thus, by using Theorem 1, the characteristic function of this process is

$$\phi_t(\theta) = e^{ib\theta t - \frac{\sigma^2\theta^2 t}{2} + i\theta\lambda t\left(\frac{p}{\eta_1 - i\theta} + \frac{d}{\eta_2 + i\theta}\right)}$$

From its Lévy density, one can check that this is, again, a finite activity process, since

$$\int_{-\infty}^{+\infty} \nu(dx) = \lambda$$

And, by differentiating the characteristic function using Theorem 2, the first two moments can be found to be

$$\mathbb{E}(L_t) = t\left(b + \lambda\left(\frac{p}{\eta_1} + \frac{d}{\eta_2}\right)\right)$$

$$Var(L_t) = t\left(\sigma^2 + \lambda\left(\frac{p}{\eta_1^2} + \frac{d}{\eta_2^2}\right)\right)$$

As we can see, both the mean and variance are finite.

3.3.3 Option Pricing

As we know, in order to get the price of an option, one has to find the present value of its final payoff. For a call option's in particular, the final payoff is $H(S_T) = (S_T - X)^+$. Then, its present value is given by

$$C_t(S_t, X, T) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[(S_T - X)\mathbb{1}_{\{S_T \geq X\}}|\mathscr{F}_t] \tag{3.6}$$

Now, introduce the notation for a probability as follows

$$\Psi(r-q,\sigma,\lambda,p,\eta_1,\eta_2,a,T) = \mathbb{Q}(Z_T \geq a|\mathscr{F}_t) \tag{3.7}$$

with $Z_t := (r-q)t + \sigma W_t + \sum_{i=1}^{N_t} Y_i$, and where Y has the double exponential distribution defined previously.

Now, by expanding equation (3.6) it follows

$$\begin{aligned} C_t(S_t,X,T) &= e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[S_T \mathbb{1}_{\{S_T \geq X\}}|\mathscr{F}_t] - e^{-r(T-t)}X\mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\{S_T \geq X\}}|\mathscr{F}_t] \\ &= e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[S_T \mathbb{1}_{\{S_T \geq X\}}|\mathscr{F}_t] - e^{-r(T-t)}X\mathbb{Q}[S_T \geq X\}|\mathscr{F}_t] \end{aligned} \tag{3.8}$$

3.3.4 Change of Numeraire

In order to compute the first expectation in the right hand side of equation (3.8), one needs to get rid of the S_T term that multiplies the indicator function. This is accomplished through a change of numeraire — see [14]. Notice that this technique is not the one used by [17]. The idea is to change to a new numeraire associated with a new martingale measure. Then, one finds the Radon-Nikodym derivative and, by comparison to the Girsanov theorem, some parameters can be extracted and, based on them, a new process can be built. Thus, let us proceed. The numeraire of the "money market account" is defined as

$$\mathbb{Q} \leftrightarrow \beta_t = \beta_0 e^{rt}$$

and the new numeraire can be defined as

$$\widetilde{\mathbb{Q}} \leftrightarrow M_t = S_t e^{qt}$$

where q refers to the asset's dividend yield. Then, the change of numeraire is done through the formula:

$$\beta_t \mathbb{E}_{\mathbb{Q}}[\frac{X_T}{\beta_T}|F_t] = M_t \mathbb{E}_{\widetilde{\mathbb{Q}}}[\frac{X_T}{M_T}|F_t]$$

where X_T represents the time-T price of any financial asset. Therefore,

$$
\begin{aligned}
C_t(S_t, X, T) &= e^{rt}\mathbb{E}^{\mathbb{Q}}[\frac{S_T}{e^{rT}}\mathbb{1}_{\{S_T \geq X\}}|\mathscr{F}_t] - e^{-r(T-t)}X\mathbb{Q}[S_T \geq X|\mathscr{F}_t] \\
&= S_t e^{qt}\mathbb{E}^{\widetilde{\mathbb{Q}}}[\frac{S_T}{S_T e^{qT}}\mathbb{1}_{\{S_T \geq X\}}|\mathscr{F}_t] - e^{-r(T-t)}X\mathbb{Q}[S_T \geq X|\mathscr{F}_t] \\
&= S_t e^{-q(T-t)}\widetilde{\mathbb{Q}}[S_T \geq X|\mathscr{F}_t] - e^{-r(T-t)}X\mathbb{Q}[S_T \geq X|\mathscr{F}_t] \\
&= S_t e^{-q(T-t)}\widetilde{\mathbb{Q}}[ln(\frac{S_T}{S_t}) \geq ln(\frac{X}{S_t})|\mathscr{F}_t] \\
&\quad - e^{-r(T-t)}X\mathbb{Q}[ln(\frac{S_T}{S_t}) \geq ln(\frac{X}{S_t})|\mathscr{F}_t] \\
&= S_t e^{-q(T-t)}\widetilde{\mathbb{Q}}[Z_T \geq ln(\frac{X}{S_t})|\mathscr{F}_t] - e^{-r(T-t)}X\mathbb{Q}[Z_T \geq ln(\frac{X}{S_t})|\mathscr{F}_t]
\end{aligned}
$$

By combining equations (3.7) and (3.5) one gets

$$
\mathbb{Q}(Z_T \geq ln(\frac{X}{S_t})|\mathscr{F}_t) = \Psi(r - q - \frac{\sigma^2}{2} - \lambda\zeta, \sigma, \lambda, p, \eta_1, \eta_2, ln(\frac{X}{S_t}), T)
$$

Now, all that is left to find is the probability under $\widetilde{\mathbb{Q}}$. Again, by the previous change of numeraire, the Radon-Nikodym is:

$$
\begin{aligned}
\frac{d\widetilde{\mathbb{Q}}}{d\mathbb{Q}} &= \frac{\frac{S_T e^{qT}}{S_t e^{qt}}}{\frac{\beta_0 e^{rT}}{\beta_0 e^{rt}}} \\
&= \frac{S_T}{S_t} e^{-(r-q)(T-t)} \\
&= e^{-(r-q)(T-t)} e^{L_{T-t}} \\
&= \underbrace{e^{-\frac{\sigma^2}{2}(T-t)+\sigma W_{T-t}}}_{\text{diffusion}} \underbrace{e^{-\lambda\zeta(T-t)+\sum_{i=1}^{N_{T-t}} Y_i}}_{\text{jump}} \qquad (3.9)
\end{aligned}
$$

3.3.5 Girsanov in Jump-Diffusion

Since one is dealing with a jump-diffusion model, an extended version of the Girsanov theorem is required.

Theorem 5. Girsanov for jump-diffusion: Let $\mathbb{Q}$ be a probability measure and $\widetilde{\mathbb{Q}}$ an equivalent martingale measure. If the Radon-Nikodym derivative is given in the form

$$\frac{d\widetilde{\mathbb{Q}}}{d\mathbb{Q}} = e^{-\frac{\sigma^2}{2}t-\sigma W_t-\lambda\zeta t+\sum_{i=1}^{N_t}(\alpha Y_i+\theta)}$$

where

$$\zeta = e^{\theta}\mathbb{E}^{\mathbb{Q}}[e^{\alpha Y}] - 1$$

then, under the new measure $\widetilde{\mathbb{Q}}$

$$\widetilde{W}_t = W_t - \sigma t$$

$$\widetilde{\lambda} = \lambda(1+\zeta)$$

$$\widetilde{f}_Y(x) = \frac{e^{\alpha x} f_Y(x)}{\int\limits_{-\infty}^{+\infty} e^{\alpha x} f_Y(x)dx}$$

being $f_Y(x)$ the distribution function of jumps and $\widetilde{f}_Y(x)$ its so called Esscher tranform — see [15].

Proof. See [9]. □

Applying Theorem 5 to equation (3.9) — with $t \to T-t$, $\alpha = 1$ and $\theta = 0$ — and following [18] and [9], the rate under the new measure is

$$\widetilde{\lambda} = \lambda(1+\zeta) = \lambda\left\{\frac{p\eta_1}{\eta_1 - 1} + \frac{p\eta_2}{\eta_2+1}\right\} = \lambda\mathbb{E}^{\mathbb{Q}}(e^Y)$$

Furthermore, following [18], the jump sizes are identical and independently distributed with density

$$\begin{aligned}
\widetilde{f}_Y(x) &= \frac{1}{\mathbb{E}^{\mathbb{Q}}(e^Y)}e^x f_Y(x) = \frac{1}{\mathbb{E}^{\mathbb{Q}}(e^Y)}e^x p\eta_1 e^{-\eta_1 x}\mathbb{1}_{\{x\geq 0\}} + \frac{1}{\mathbb{E}^{\mathbb{Q}}(e^Y)}e^x d\eta_2 e^{\eta_2 x}\mathbb{1}_{\{x<0\}} \\
&= p\left(\frac{p\eta_1}{\eta_1-1}+\frac{p\eta_2}{\eta_2+1}\right)^{-1} e^x \frac{\eta_1}{\eta_1-1}(\eta_1-1)e^{-(\eta_1-1)x}\mathbb{1}_{\{x\geq 0\}} \\
&+ d\left(\frac{p\eta_1}{\eta_1-1}+\frac{p\eta_2}{\eta_2+1}\right)^{-1} e^x \frac{\eta_2}{\eta_2+1}(\eta_2+1)e^{-(\eta_2+1)x}\mathbb{1}_{\{x<0\}} \\
&= \widetilde{p}\widetilde{\eta}_1 e^{-\widetilde{\eta}_1 x}\mathbb{1}_{\{x\geq 0\}} + \widetilde{d}\widetilde{\eta}_2 e^{-\widetilde{\eta}_2 x}\mathbb{1}_{\{x<0\}}
\end{aligned}$$

where

$$\widetilde{p} = \left(\frac{p\eta_1}{\eta_1 - 1} + \frac{d\eta_2}{\eta_2 + 1}\right)^{-1} \frac{\eta_1}{\eta_1 - 1}$$

$$\widetilde{d} = \left(\frac{p\eta_1}{\eta_1 - 1} + \frac{d\eta_2}{\eta_2 + 1}\right)^{-1} \frac{\eta_2}{\eta_2 + 1}$$

$$\widetilde{\eta_1} = \eta_1 - 1 \ , \ \widetilde{\eta_1} = \eta_2 + 1$$

Then

$$\widetilde{\mathbb{Q}}(Z_T \geq ln(\frac{X}{S_t})|\mathscr{F}_t) = \Psi(r - q + \frac{\sigma^2}{2} - \lambda\zeta, \sigma, \widetilde{\lambda}, \widetilde{p}, \widetilde{\eta_1}, \widetilde{\eta_2}, ln(\frac{X}{S_0}), T)$$

and finally

$$\begin{aligned} C_t(S_t, X, T) =& e^{-r(T-t)} S_t \Psi(r - q + \frac{\sigma^2}{2} - \lambda\zeta, \sigma, \widetilde{\lambda}, \widetilde{p}, \widetilde{\eta_1}, \widetilde{\eta_2}, ln(\frac{X}{S_t}), T) \\ &- e^{-r(T-t)} \Psi(r - q - \frac{\sigma^2}{2} - \lambda\zeta, \sigma, \lambda, p, \eta_1, \eta_2, ln(\frac{X}{S_t}), T) \end{aligned}$$

3.3.6 Pricing Formula

In order to achieve the pricing formula, the probabilities of exercise Ψ need to be specified. In order to do so, it is necessary to find the density for the sum of a normal random variable and a double exponential. The derivation is extensive and can be found in [17]. To specify the probabilities, first it is necessary to define some special functions:

$$Hh_{-1}(x) = e^{-\frac{x^2}{2}}$$

$$Hh_0(x) = \sqrt{2\pi}\Phi(-x)$$

$$Hh_n(x) = \int_x^{+\infty} Hh_{n-1}(y)dy = \frac{1}{n!}\int_x^{+\infty} (t - x)^n e^{-\frac{t^2}{2}} dt$$

$$I_n(c, \alpha, \beta, \gamma) = \int_c^{+\infty} e^{\alpha x} Hh_n(\beta c - \gamma)dx \quad \forall n \geq 1$$

where Φ is the standard normal cumulative distribution. Then

$$nHh_n(x) = Hh_{n-2}(x) - xHh_{n-1}(x) \quad \forall n \geq 1$$

and $\forall n \geq -1$

$$I_n(c, \alpha, \beta, \gamma) = -\frac{e^{\alpha c}}{\alpha}\left(\frac{\beta}{\alpha}\right)^{n-1} Hh_i(\beta c - \gamma) + \left(\frac{\beta}{\alpha}\right)^{n+1} \frac{\sqrt{2\pi}}{\beta} e^{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}} \Phi\left(\theta - \beta c + \gamma + \frac{\alpha}{\delta}\right)$$

with

$$\theta = \begin{cases} 1 & \Leftarrow \ \beta > 0, \ \alpha \neq 0 \\ -1 & \Leftarrow \ \beta < 0, \ \alpha \leq 0 \end{cases}$$

Finally, the exercise probability is defined as

$$\begin{aligned}
&\Psi(\alpha, \sigma, \lambda, p, \eta_1, \eta_2, a, T) \\
= \ & \frac{e^{(\sigma\eta_1)^2 \frac{T}{2}}}{\sigma\sqrt{2\pi T}} \sum_{n=1}^{+\infty} \pi_n \sum_{k=1}^{n} P_{n,k}(\sigma\sqrt{T}\eta_1)^k \times I_{k-1}(a - \alpha T; -\eta_1; -\frac{1}{\sigma\sqrt{T}}; -\sigma\eta_1\sqrt{T}) \\
&+ \frac{e^{(\sigma\eta_2)^2 \frac{T}{2}}}{\sigma\sqrt{2\pi T}} \sum_{n=1}^{+\infty} \pi_n \sum_{k=1}^{n} D_{n,k}(\sigma\sqrt{T}\eta_2)^k \times I_{k-1}(a - \alpha T; -\eta_2; -\frac{1}{\sigma\sqrt{T}}; -\sigma\eta_2\sqrt{T}) \\
&+ \pi_0 \Phi(-\frac{a - \alpha}{\sigma\sqrt{T}})
\end{aligned}$$

where

$$P_{n,i} := \sum_{j=1}^{n-1} p^j d^{n-j} \binom{n-i-1}{j-i} \left(\frac{\eta_1}{\eta_1 + \eta_2}\right)^{j-i} \left(\frac{\eta_2}{\eta_1 + \eta_2}\right)^{n-j} \quad , \quad 1 \leq i \leq n-1$$

$$D_{n,i} := \sum_{j=1}^{n-1} d^j p^{n-j} \binom{n-i-1}{j-i} \left(\frac{\eta_2}{\eta_1 + \eta_2}\right)^{j-i} \left(\frac{\eta_1}{\eta_1 + \eta_2}\right)^{n-j} \quad , \quad 1 \leq i \leq n-1$$

$$P_{n,n} := p^n \qquad D_{n,n} := d^n \qquad \pi_n = \frac{e^{-\lambda T}\lambda^n}{n!}$$

Chapter 4

Pure Jumps

4.1 The Idea

Up until now we have been dealing with finite activity models, namely jump-diffusions. Now the idea is to use a jump process with infinite activity. Unlike the previous models, these processes generate purely discontinuous paths, e.g., there is no diffusive component. Instead of having a drifted Brownian motion generating the paths and, once in a while, a jump occurring, the paths are generated by infinite jumps, most of them small.

Moreover, a totaly different valuation approach will be used. In the previous models, closed form solutions were obtained for pricing European-style options. In the Merton model the solution is expressed in terms of Black-Scholes values and in Kou is expressed by using special mathematical functions. But a closed form solution is not always possible to obtain and, in some cases, for instance in the Kou model, it requires a laborious mathematical treatment. For a more general exponential Lévy process a stronger formalism is required, and the use of characteristic functions and numerical integration turns out to be the most cost-effective procedure for pricing European-style options.

4.2 The Variance Gamma Model

4.2.1 Dynamics

The Variance Gamma process — see [20] — is a three parameter generalization of the Brownian motion and is obtained by evaluating a Brownian motion with constant drift and volatility at a random time change given by a Gamma process, with constant drift and volatility. Each unit of calendar time can be considered as having an economically relevant time length given by an independent random variable which has a density with unit mean and positive variance. Under the Variance Gamma process the unit period continuously compounded return is normally distributed, conditional on the realization of a random time change. In addition to the volatility of the Brownian motion there are two additional parameters that control both kurtosis and skewness. The Variance Gamma process has no continuous martingale component, contrary to the previous models. It is a pure jump process with infinite activity. See also [21]. The dynamics for the log returns proposed by [20] is:

$$L_t = ln\left(\frac{S_t}{S_0}\right) = (r-q)t + Y_t(\sigma, \theta, \nu) + \omega t \tag{4.1}$$

and ω is set to be $\omega = \frac{1}{\nu} ln(1 - \theta\nu - \frac{\sigma^2\nu}{2})$ so that the expectation of the log returns equals r, the risk free rate. Notice that there is a drift $(r-q)t$, a pure jump component Y_t and ωt plays the role of a compensator, in analogy with the previous models. However, one question remains. What is Y_t?

Consider a drifted Brownian motion $b_t(\mu, \sigma) = \theta t + \sigma W_t$. Now, replace time by a Gama process with mean rate $\theta = 1$ and variance rate ν, so $t \to \gamma_t(1, \nu)$. This procedure, replacing time with a stochastic process, is called subordination. Then define $Y_t(\sigma, \theta, \nu) = b_{\gamma_t(1,\nu)}(\theta, \sigma)$ as a Variance Gamma process. Replacing time by a Gamma process is the way of giving every unit of time an economical relevant time length, as described earlier.

So, equation (4.1) indicates that the log returns on the asset follow a drifted Variance Gamma motion.

4.2.2 Option Pricing

Assuming that the characteristic function of a random variable — say $ln(S_T)$ — is known analytically, [5] provide a formula to numerically determine the price of an European-style option.

Theorem 6. Under the Variance Gamma model, the price of an European-style call option is given by

$$C_t(S_t, X, T) = S_t e^{-q(T-t)}\Pi_1 - e^{-r(T-t)}X\Pi_2$$

where

$$\Pi_1 = \widetilde{Q}(S_T > X) = \frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty} Re\left(\frac{e^{-iuln(X)}\phi_{ln(S_T)}(u)}{iu}\right) du$$

$$\Pi_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty} Re\left(\frac{e^{-iuln(X)}\phi_{ln(S_T)}(u-i)}{iu\phi_{ln(S_T)}(-i)}\right) du$$

The price of an European-style put option can be calculated through put-call parity.

The integrals in Theorem 6 can be calculated numerically, using, for example, a Gaussian quadrature — see, for instance, [26].

4.2.3 Characteristic Function

According to [20], the characteristic function of a Variance Gamma process is given by

$$\phi_{Y_t}(u) = \mathbb{E}[e^{iuY_t}] = \left(\frac{1}{1 - iu\theta\nu + \frac{\sigma^2\nu}{2}u^2}\right)^{\frac{t}{\nu}}$$

However, one needs the characteristic function for the log returns. We have defined previously that

$$L_t = ln\left(\frac{S_t}{S_0}\right)$$

and

$$ln(S_t) = ln(S_0) + (r - q + \omega)t + Y_t$$

Thus,

$$\begin{aligned}
\phi_{ln(S_t)} &= \mathbb{E}[e^{iuln(S_t)}] \\
&= \mathbb{E}[e^{iu(ln(S_0)+(r-q+w)t+Y_t)}] \\
&= \mathbb{E}[e^{iu(ln(S_0)+(r-q+w)t)}]\mathbb{E}[e^{iuY_t}] \\
&= e^{iu(ln(S_0)+(r-q)t+\frac{t}{\nu}ln(1-\theta\nu-\frac{\sigma^2\nu}{2}))}\phi_{Y_t}(u)
\end{aligned}$$

See also [19]. Now, all that is left to do is to apply Theorem 6.

Thus, it is very easy to price European-style options using this formalism. One first gets the characteristic function of the log returns and then applies Theorem 6. As simple as that. The disadvantage of this method is that it requires a numerical integration. Nevertheless, it is the most cost-effective formalism. Notice that in the Merton and Kou models, the value of an European-style option could as well be found using this method, and it would have been quite simpler. For the purpose, one would have to use the characteristic functions of the respective models.

One could ask: what about using Fast Fourier Transform (FFT) in order to calculate the integrals? In fact, FFT is a very fast method. Its application to option pricing can be found in [8]. However, there are certain stability conditions to be taken into account, and these are not as simple as [8] states. A more detailed study on the stability conditions for FFT and comparison with other methods can be found in [2].

Chapter 5

Monte Carlo

Another way to price a contingent claim is through Monte Carlo simulations. There is no need for using Monte Carlo simulations in the previous models, since there are closed form solutions, which are much quicker to compute. Nevertheless, a demonstration of this method will be performed, since it can be very useful on more complex contexts where there is no closed form solution and because it is important to show how to simulate paths from the stochastic differential equations that define the asset's dynamics. The idea is to simulate the paths of the asset's price, averaging the final payoff for a large number of simulations and then discount it using the risk-free rate.

5.1 Merton

For the Merton model, the paths can be simulated through the following equation:

$$ln(S_{t+\Delta t}) = ln(S_t) + (r - q - \frac{\sigma^2}{2} - \lambda\zeta)\Delta t + \sigma\epsilon\sqrt{\Delta t} + J$$

where

$$J = \begin{cases} u \sim \mathcal{N}(\mu, \delta^2), & \text{with probability } \lambda\Delta t \\ 0, & \text{with probability } 1 - \lambda\Delta t \end{cases}$$

which is a Bernoulli variable, and $\epsilon \sim N(0,1)$. Notice that the Bernoulli variable works as an asymptotical approximation for the arrival of jumps in the Poisson process.

See some simulations of this model in Appendix A.3.

5.2 Kou

As for Kou model, the method is very similar. However, the jump component is different. It can be modeled as

$$J = \begin{cases} \begin{cases} u \sim \eta_1 e^{-\eta_1 x} \mathbf{1}_{\{x \geq 0\}}, & \text{with probability } p \\ v \sim \eta_2 e^{\eta_2 x} \mathbf{1}_{\{x < 0\}}, & \text{with probability } d \end{cases} & \text{with probability } \lambda \Delta t \\ 0 & \text{with probability } 1 - \lambda \Delta t \end{cases}$$

5.3 Variance Gamma

The paths can be simulated through:

$$ln(S_{t+\Delta t}) = ln(S_t) + (r - q + \omega)\Delta t + \theta G_{t+\Delta t} + \sigma \epsilon \sqrt{G_{t+\Delta t}}$$

where

$$G \sim \Gamma\left(\frac{t + \Delta t}{\nu}, \nu\right)$$

Γ being the gamma distribution. See, for instance, [13].

Chapter 6

Calibration

6.1 Method

Now it is time to test the models and to check if they fit the real data and what advantages they bring over the traditional diffusion models. In order to so, one shall minimize the sum of the squared percentage errors for a given set of N options:

$$\min_{v} \sum_{i=1}^{N} \left(\frac{\text{ModelPrice}_i(v) - \text{MarketPrice}_i}{\text{MarketPrice}_i} \right)^2$$

The data contains options on stock and stock market indices, namely, Apple, Nikkei, Ibovespa, FTSE and Nasdaq. Additionally, local and stochastic volatility models with no jump component are calibrated, namely the CEV — see [11] — and Heston — see [16] — models, so one can compare them all and see what advantages may come from each of them.

6.2 Results

The results are from Appendix B.1. They are presented as MAPE — Mean Average Percentage Error — so that it is easier to check the quality of the fitting. The parameters can be found in Appendix B.2 and the corresponding options in Appendix C.

6.3 Analysis

In all cases some model turns out to fit the data better than the Black-Scholes model. However it is not always the same model that brings better results and sometimes the improvement is not that significant. This happens because the models are constructed assuming that the market "thinks" that a given asset price follows a determined stochastic differential equation, which may not be the most correct one. For different underlying assets the market might have different assumptions. One must keep in mind that one is trying to create a model that may explain how the market prices the options, independently of what one might think about a given asset's historical behavior, although the market should act mainly according to that, at least in theory. There is no model which is absolutely the most efficient, however, in some cases the stochastic volatility models show better results, though they have no jump component. So, the fact that the volatility is constant in time as well as the rate arrival of jumps is a limitation to be taken into account in the models studied, since it is unrealistic to think that the market implicitly assumes the same volatility and the same rate arrival of jumps for every moment in time.

The optimization algorithm is something to be considered as well, since, as one can see from Appendix B.1, it is possible to find different minima using different algorithms. It is also important to notice that the errors in the fitting arise not only from the limitations of the models alone, but also from the possibility of non verification of certain basic conditions that are widely assumed in financial modeling, for instance, the non existence of arbitrage, the fact that one can buy or (short) sell anything at anytime, the assumption that there are no transaction costs or a bid-ask spread. There can also be the problem that when the prices are extracted, they may not be up to date with the spot, due to some lag between the pricing of the derivative by the market and the spot price.

Nevertheless, one needs to say that the results, although not perfect, they bring some improvements over the traditional Black-Scholes model.

Chapter 7

Conclusions

7.1 Remarks

In this thesis several models and methods for pricing European-style options were exposed. One started by introducing some important methods on stochastic calculus with jumps. Then, three models were presented: the Merton, the Kou and the Variance Gamma. Two of them, the Merton and the Kou, are finite activity jump-diffusions. The Variance Gamma, though, is an infinite activity pure jump model.

For each model, a different pricing approach was performed. In the Merton case, the options' prices were given by a weighted average of Black-Scholes prices. In the Kou model, the solution was given in terms of special mathematical functions and, in the Variance Gamma model, a semi-closed form solution was obtained, though quite simple and understandable.

As for the methods used, in the Merton model one expressed the price of an option in terms of Black-Scholes prices by conditioning on the number of jumps. This method is applicable for the Merton model alone and gives closed form solutions for European-style options only. In the Kou model, one recurred to a change of numeraire technique and to a Girsanov theorem for jump-diffusions. Moreover, complicated mathematical functions were required in order to define the probabilities of exercise. However, although only European-style options were studied, it is important to remark that the Kou model presents closed form solutions for the majority of option styles. In the

Variance-Gamma model, one used characteristic functions and numerical integration, in order to perform a Fourier inversion. From my point of view, the formalism applied to the Variance Gamma model is the most cost-effective, since it only requires the characteristic function of a process, which, in the case of Lévy processes, can be easily built via the Lévy-Khintchine representation. This is a great advantage of the Lévy process with respect to, for instance, the stochastic volatility models, since for the latter the computation of the characteristic function can be much more complicated, for the increments of the process are correlated, which is not verified in Lévy processes. Then, it was shown how these models' paths can be simulated and how, therefore, the options can be priced via Monte-Carlo simulations. It was not necessary to apply it to our cases, since closed form solutions were obtained. Nevertheless it is an important methodology that can be applied under more complicated contexts.

Finally, the models were calibrated with real data from the options' market, and we saw how well these models fit the market for the given cases. No model showed to be absolutely the best. However, all of them brought improvements with respect to the traditional pure diffusion Black-Scholes model.

In this work, I also provide Matlab® code for the valuation formulae, which may be very useful to the reader. It can be found in Appendix D.

Thus, one can conclude that the Lévy processes bring together some improvements in explaining the market as well as a variety of solid frameworks to achieve pricing formulae.

7.2 Further Reading

For those interested in investigating even further, I want to suggest some reading. It would be interesting to extend these models, and study models that combine both jumps and stochastic volatility. There is a model proposed by [6] that combines the Merton jump-diffusion model and the Heston stochastic volatility model, by defining two stochastic differential equations: one for the asset's price and another for the volatility associated with the Brownian motion. Another interesting model is one that also includes a stochastic differential equation for the rate arrival of jumps — see [12]. Since stochastic volatility implies correlation between the increments of the process, these models lie beyond the scope of Lévy processes.

With respect to methodology, an interesting field is the option valuation approach via partial integro-differential equations and their solution through finite difference methods — see for instance, [27].

7.3 Final Note

This is a field with lots of research yet to be done, and I hope that with the reading of this thesis the reader may feel motivated to move to further investigation.

Appendix A

Graphics

A.1 Empirical Evidence

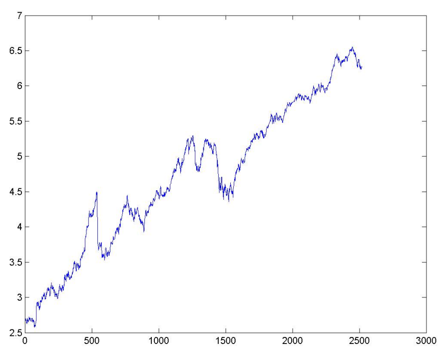


Figure A-1: Apple's log prices

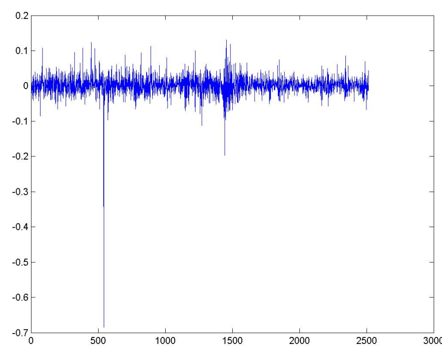


Figure A-2: Apple's log-returns

Figure A-1 shows the daily logarithm of Apple's stock price between 01/01/2003 and 01/01/2013. Figure A-2 shows the daily logarithmic returns on Apple's stock price between 01/01/2003 and 01/01/2013.

Large variations can be better seen in Figure A-2. Notice that although most of the values lie in a small region, sometimes some spikes show up. These represent large variations, which we call jumps.

A.2 Merton Prices

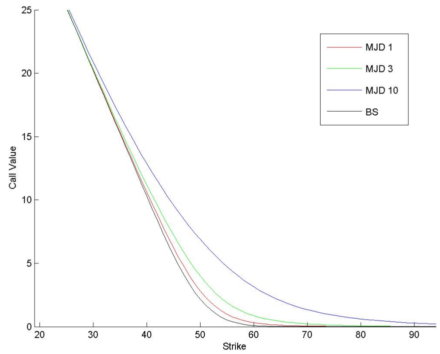


Figure A-3: Merton Model: Value vs Strike - varying lambda

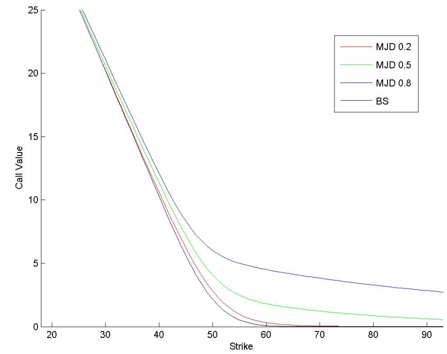


Figure A-4: Merton Model: Value vs Strike - varying delta

Figure A-3 shows the value of a call option as a function of the strike, varying the parameter λ of the Merton model, keeping the other parameters fixed. The parameters used are: $\lambda \in \{1, 3, 10\}$, $\mu = -0.1$ and $\delta = 0.1$. Figure A-4 shows the value of a call option as a function of the strike, varying the parameter δ of the Merton model, keeping the other parameters fixed. The parameters used are: $\lambda = 1$, $\mu = -0.1$ and $\delta \in \{0.2, 0.5, 0.8\}$.

As we can check, the value of the option under the Merton model relative to the value under the Black-Scholes model increases as the rate arrival of jumps λ increases and as the variance of the jump distribution δ increases. This was expected, since more jumps with greater variance mean more uncertainty in the expected final payoff.

A.3 Path Simulation

Figure A-5 shows a simulation of the logarithm of the stock price under the Merton model for 360 days. Figure A-6 shows the first differences of the logarithm of stock the price under the Merton model for 360 days. Figure A-7 shows a simulation of the logarithm of the stock price under the Black-Scholes model for 360 days. Figure A-8 shows the first differences of the logarithm of the stock price under the Black-Scholes

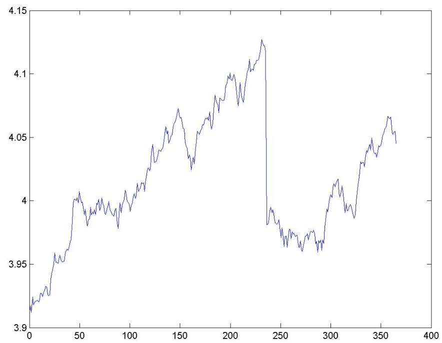


Figure A-5: Merton model path simulation

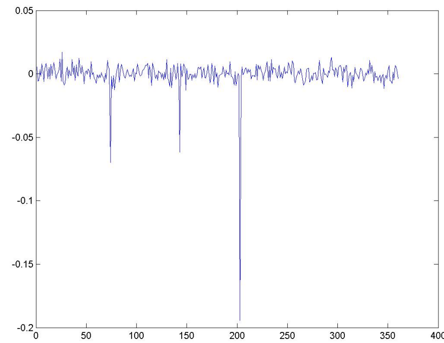


Figure A-6: Returns for Merton model path simulation

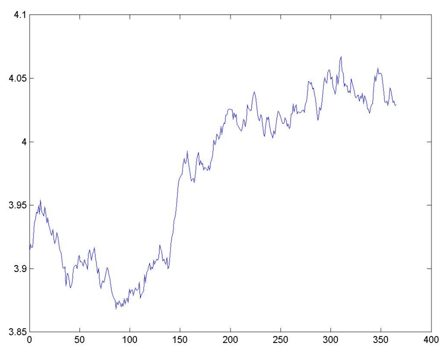


Figure A-7: Black-Scholes model path simulation

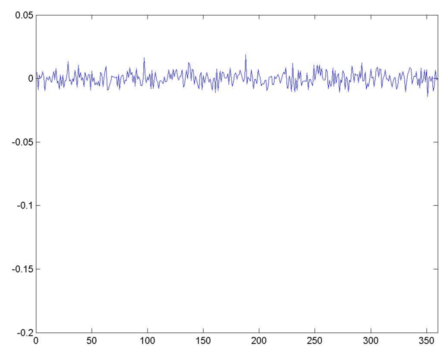


Figure A-8: Returns for Black-Scholes model path simulation

model for 360 days.

It is an empirical evidence that large variations do happen. By comparing Figures A-5 and A-6 with Figures A-7 and A-8 we can check that the inclusion of jumps brings indeed the likeliness of large variations in an asset price. It can be seen more clearly from Figures A-5 and A-7. Notice that in Figure A-5 all the values lie in a tiny interval. In Figure A-7 most of the values are places in a tiny interval as well. However, we see some spikes of large variation. Thus, if we compare both simulations to Figures A-1 and A-2 we can clearly see that the model that admits jumps — the Merton model — is more realistic than the Black-Scholes diffusion model.

Appendix B

Calibration Results

B.1 MAPE

Apple 14/06/2013		
Model	MAPE	
BS	8.79%	
MJD	3.18%	(*)(**)
Kou	8.79%	
VG	5.64%	
CEV	8.82%	
Heston	6.55%	

Apple(2) 23/10/2012		
Model	MAPE	
BS	5.47%	
MJD	3.83%	(**)
Kou	5.47%	
VG	3.20%	(*)
CEV	5.22%	
Heston	5.37%	

Ibovespa 01/07/2013		
Model	MAPE	
BS	4.24%	
MJD	4.22%	(**)
Kou	5.00%	(**)
VG	3.75%	
CEV	4.12%	
Heston	3.21%	(*)

FTSE 01/07/2013		
Model	MAPE	
BS	19.98%	
MJD	19.98%	
Kou	18.92%	(*)(**)
VG	20.07%	
CEV	19.66%	
Heston	20.90%	

Nasdaq	01/07/2013	
Model	MAPE	
BS	14.06%	
MJD	6.89%	(*)(**)
Kou	14.06%	
VG	11.58%	
CEV	11.98%	
Heston	14.24%	

Nikkei	24/06/2013	
Model	MAPE	
BS	7.72%	
MJD	7.72%	
Kou	7.47%	(**)
VG	7.88%	
CEV	7.67%	(*)
Heston	7.84%	

(*) model with the best fitting.

(**) result was obtained through simulated annealing algorithm, instead of Matlab®'s fminsearch algorithm used by default.

The corresponding options can be found in Appendix C.

B.2 Parameters

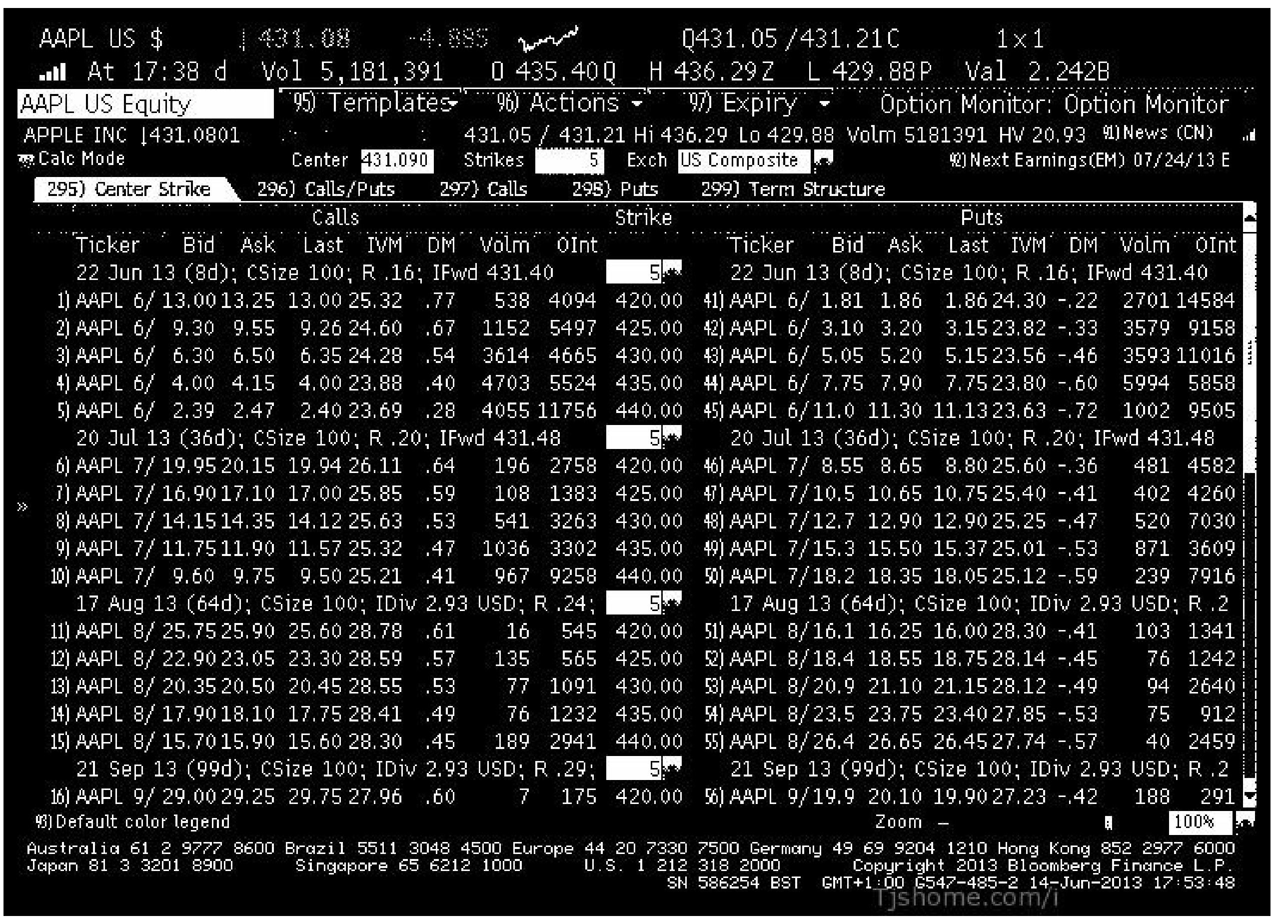
Merton	δ	μ	λ	σ
Apple	0.8651	-0.4707	0.3318	0.1775
Apple(2)	0.1996	-11.4801	0.0882	0.2217
Ibovespa	0.8063	-8.6553	0.0000	0.1918
FTSE	0.0245	0.0138	0.0000	0.1486
Nasdaq	0.6672	-1.5696	0.0602	0.1227
Nikkei	0.0213	0.0286	0.0000	0.3444

Kou	σ	λ	p	η_1	η_2
Apple	0.2453	0.0009	0.5603	1.6889	0.5134
Apple(2)	0.2640	0.0010	0.5607	1.6600	0.5068
Ibovespa	0.1920	0.0000	0.5929	1.5299	0.5031
FTSE	0.1486	0.0000	0.5981	1.5239	0.5097
Nasdaq	0.1542	0.0000	0.5892	1.5247	0.4977
Nikkei	0.3459	0.6108	0.0756	18.8657	4.3408

Variance Gamma	θ	ν	σ
Apple	0.0378	0.0583	0.2745
Apple(2)	-0.1372	0.1053	0.2925
Ibovespa	1.0000	0.0172	0.1390
FTSE	-0.9077	0.0187	0.0856
Nasdaq	-0.4623	0.1024	0.0966
Nikkei	-4.5027	0.0061	0.0152

Appendix C

Data



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At 17:38 d Vol 5,181,391 O 435.40Q H 436.29Z L 429.88P Val 2.242B

AAPL US Equity 95) Templates 96) Actions 97) Expiry Option Monitor: Option Monitor

APPLE INC 431.0801 431.05 / 431.21 Hi 436.29 Lo 429.88 Volm 5181391 HV 20.93 91) News (CN)

Calc Mode Center 431.090 Strikes 5 Exch US Composite 92) Next Earnings(EM) 07/24/13 E

295) Center Strike 296) Calls/Puts 297) Calls 298) Puts 299) Term Structure

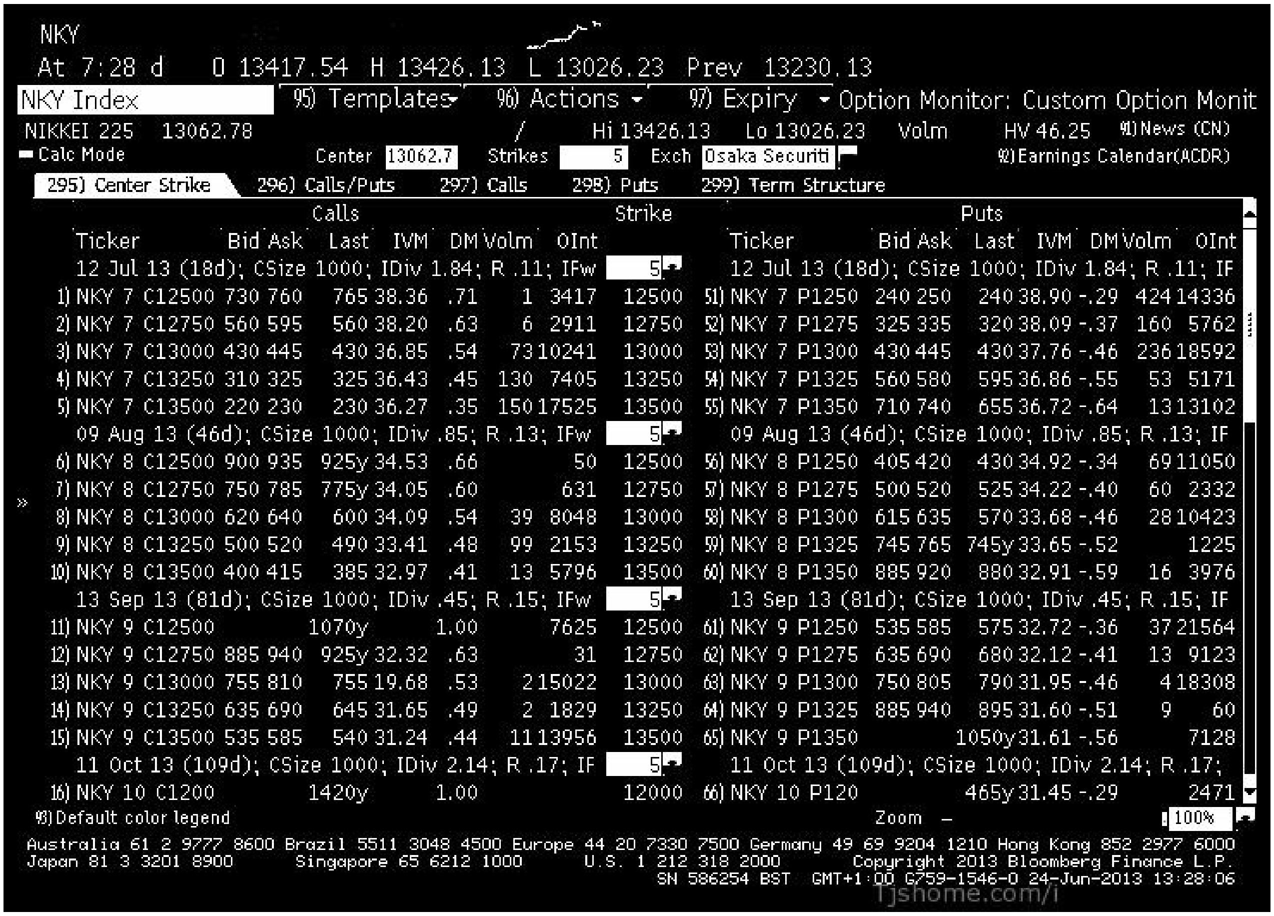
Calls								Strike	Puts							
Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt		Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
22 Jun 13 (8d); CSize 100; R .16; IFwd 431.40								5	22 Jun 13 (8d); CSize 100; R .16; IFwd 431.40							
1) AAPL 6/	13.00	13.25	13.00	25.32	.77	538	4094	420.00	41) AAPL 6/	1.81	1.86	1.86	24.30	-.22	2701	14584
2) AAPL 6/	9.30	9.55	9.26	24.60	.67	1152	5497	425.00	42) AAPL 6/	3.10	3.20	3.15	23.82	-.33	3579	9158
3) AAPL 6/	6.30	6.50	6.35	24.28	.54	3614	4665	430.00	43) AAPL 6/	5.05	5.20	5.15	23.56	-.46	3593	11016
4) AAPL 6/	4.00	4.15	4.00	23.88	.40	4703	5524	435.00	44) AAPL 6/	7.75	7.90	7.75	23.80	-.60	5994	5858
5) AAPL 6/	2.39	2.47	2.40	23.69	.28	4055	11756	440.00	45) AAPL 6/	11.0	11.30	11.13	23.63	-.72	1002	9505
20 Jul 13 (36d); CSize 100; R .20; IFwd 431.48								5	20 Jul 13 (36d); CSize 100; R .20; IFwd 431.48							
6) AAPL 7/	19.95	20.15	19.94	26.11	.64	196	2758	420.00	46) AAPL 7/	8.55	8.65	8.80	25.60	-.36	481	4582
7) AAPL 7/	16.90	17.10	17.00	25.85	.59	108	1383	425.00	47) AAPL 7/	10.5	10.65	10.75	25.40	-.41	402	4260
8) AAPL 7/	14.15	14.35	14.12	25.63	.53	541	3263	430.00	48) AAPL 7/	12.7	12.90	12.90	25.25	-.47	520	7030
9) AAPL 7/	11.75	11.90	11.57	25.32	.47	1036	3302	435.00	49) AAPL 7/	15.3	15.50	15.37	25.01	-.53	871	3609
10) AAPL 7/	9.60	9.75	9.50	25.21	.41	967	9258	440.00	50) AAPL 7/	18.2	18.35	18.05	25.12	-.59	239	7916
17 Aug 13 (64d); CSize 100; IDiv 2.93 USD; R .24;								5	17 Aug 13 (64d); CSize 100; IDiv 2.93 USD; R .2							
11) AAPL 8/	25.75	25.90	25.60	28.78	.61	16	545	420.00	51) AAPL 8/	16.1	16.25	16.00	28.30	-.41	103	1341
12) AAPL 8/	22.90	23.05	23.30	28.59	.57	135	565	425.00	52) AAPL 8/	18.4	18.55	18.75	28.14	-.45	76	1242
13) AAPL 8/	20.35	20.50	20.45	28.55	.53	77	1091	430.00	53) AAPL 8/	20.9	21.10	21.15	28.12	-.49	94	2640
14) AAPL 8/	17.90	18.10	17.75	28.41	.49	76	1232	435.00	54) AAPL 8/	23.5	23.75	23.40	27.85	-.53	75	912
15) AAPL 8/	15.70	15.90	15.60	28.30	.45	189	2941	440.00	55) AAPL 8/	26.4	26.65	26.45	27.74	-.57	40	2459
21 Sep 13 (99d); CSize 100; IDiv 2.93 USD; R .29;								5	21 Sep 13 (99d); CSize 100; IDiv 2.93 USD; R .2							
16) AAPL 9/	29.00	29.25	29.75	27.96	.60	7	175	420.00	56) AAPL 9/	19.9	20.10	19.90	27.23	-.42	188	291

93) Default color legend Zoom – 100%

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2013 Bloomberg Finance L.P.
SN 586254 BST GMT+1:00 G547-485-2 14-Jun-2013 17:53:48

Tjshome.com/i

Figure C-1: Options on Apple's



NKY
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NKY Index 95) Templates 96) Actions 97) Expiry Option Monitor: Custom Option Monit
NIKKEI 225 13062.78 / Hi 13426.13 Lo 13026.23 Volm HV 46.25 91) News (CN)
Calc Mode Center 13062.7 Strikes 5 Exch Osaka Securiti 92) Earnings Calendar(ACDR)
295) Center Strike 296) Calls/Puts 297) Calls 298) Puts 299) Term Structure

Calls Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt	Strike	Puts Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
12 Jul 13 (18d); CSize 1000; IDiv 1.84; R .11; IFw								5	12 Jul 13 (18d); CSize 1000; IDiv 1.84; R .11; IF							
1) NKY 7 C12500	730	760	765	38.36	.71	1	3417	12500	51) NKY 7 P1250	240	250	240	38.90	-.29	424	14336
2) NKY 7 C12750	560	595	560	38.20	.63	6	2911	12750	52) NKY 7 P1275	325	335	320	38.09	-.37	160	5762
3) NKY 7 C13000	430	445	430	36.85	.54	73	10241	13000	53) NKY 7 P1300	430	445	430	37.76	-.46	236	18592
4) NKY 7 C13250	310	325	325	36.43	.45	130	7405	13250	54) NKY 7 P1325	560	580	595	36.86	-.55	53	5171
5) NKY 7 C13500	220	230	230	36.27	.35	150	17525	13500	55) NKY 7 P1350	710	740	655	36.72	-.64	13	13102
09 Aug 13 (46d); CSize 1000; IDiv .85; R .13; IFw								5	09 Aug 13 (46d); CSize 1000; IDiv .85; R .13; IF							
6) NKY 8 C12500	900	935	925y	34.53	.66		50	12500	56) NKY 8 P1250	405	420	430	34.92	-.34	69	11050
7) NKY 8 C12750	750	785	775y	34.05	.60		631	12750	57) NKY 8 P1275	500	520	525	34.22	-.40	60	2332
8) NKY 8 C13000	620	640	600	34.09	.54	39	8048	13000	58) NKY 8 P1300	615	635	570	33.68	-.46	28	10423
9) NKY 8 C13250	500	520	490	33.41	.48	99	2153	13250	59) NKY 8 P1325	745	765	745y	33.65	-.52		1225
10) NKY 8 C13500	400	415	385	32.97	.41	13	5796	13500	60) NKY 8 P1350	885	920	880	32.91	-.59	16	3976
13 Sep 13 (81d); CSize 1000; IDiv .45; R .15; IFw								5	13 Sep 13 (81d); CSize 1000; IDiv .45; R .15; IF							
11) NKY 9 C12500			1070y		1.00		7625	12500	61) NKY 9 P1250	535	585	575	32.72	-.36	37	21564
12) NKY 9 C12750	885	940	925y	32.32	.63		31	12750	62) NKY 9 P1275	635	690	680	32.12	-.41	13	9123
13) NKY 9 C13000	755	810	755	19.68	.53	2	15022	13000	63) NKY 9 P1300	750	805	790	31.95	-.46	4	18308
14) NKY 9 C13250	635	690	645	31.65	.49	2	1829	13250	64) NKY 9 P1325	885	940	895	31.60	-.51	9	60
15) NKY 9 C13500	535	585	540	31.24	.44	11	13956	13500	65) NKY 9 P1350			1050y	31.61	-.56		7128
11 Oct 13 (109d); CSize 1000; IDiv 2.14; R .17; IF								5	11 Oct 13 (109d); CSize 1000; IDiv 2.14; R .17;							
16) NKY 10 C1200			1420y		1.00			12000	66) NKY 10 P120			465y	31.45	-.29		2471

93) Default color legend Zoom – 100%
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Figure C-2: Options on Nikkei

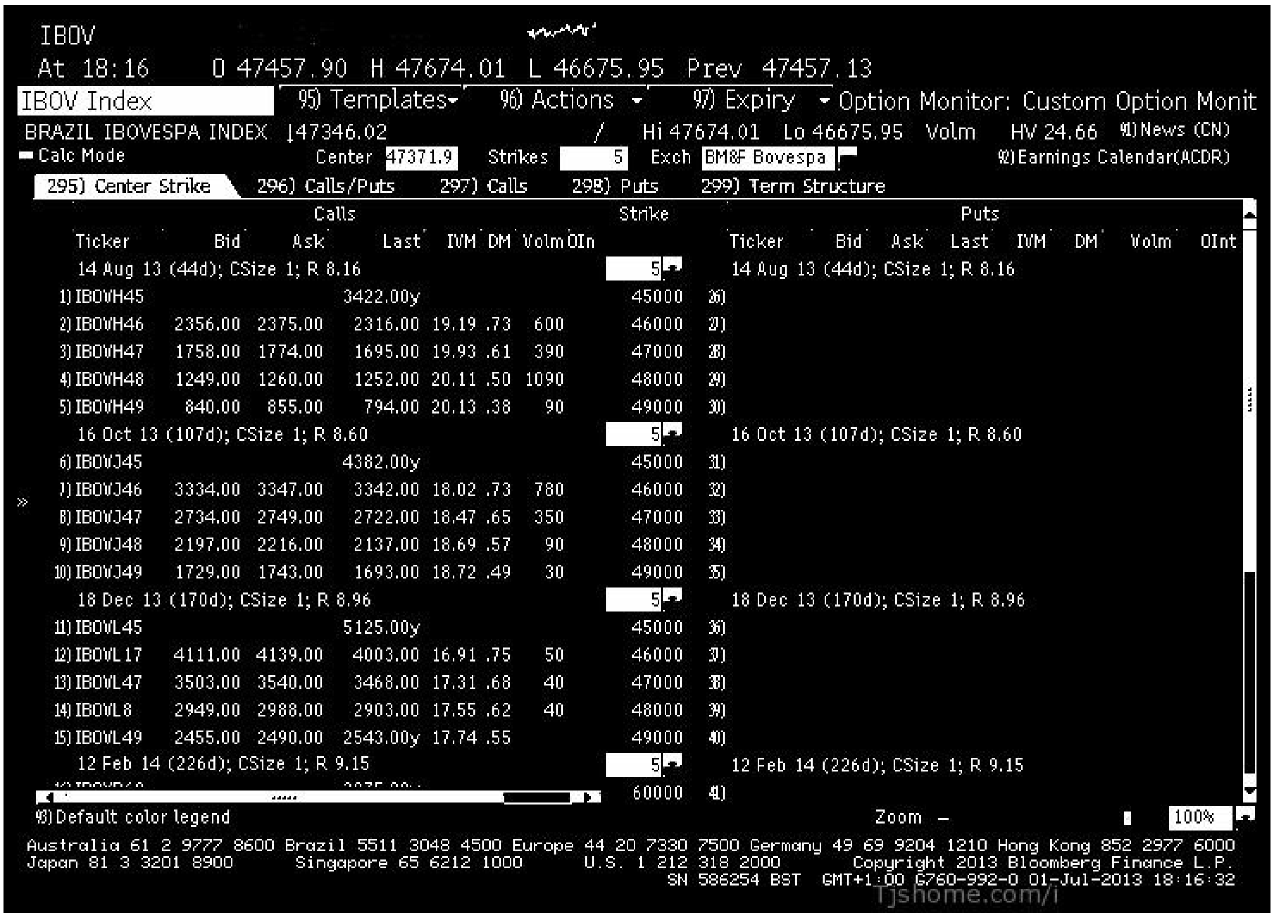
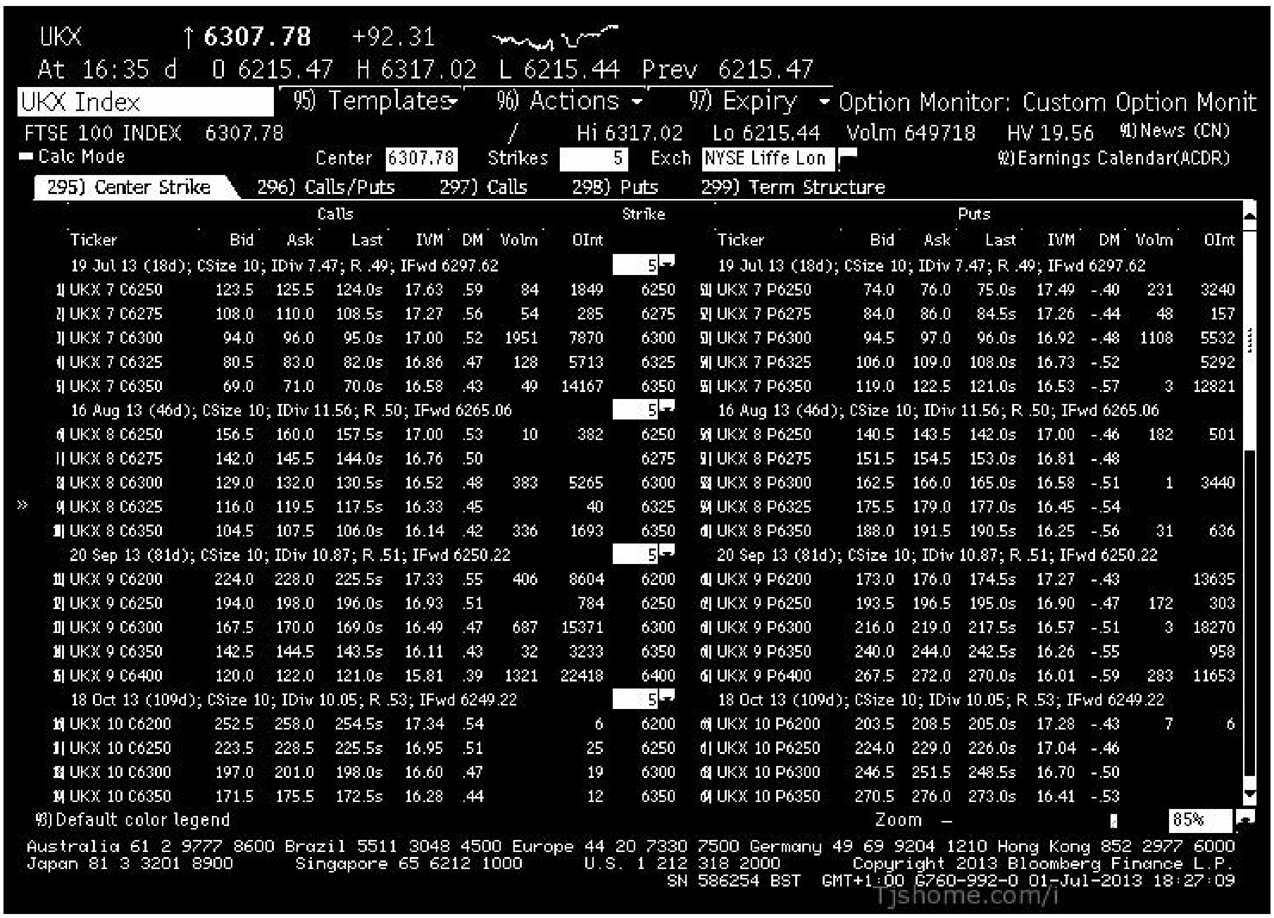


Figure C-3: Options on Ibovespa



UKX ↑ 6307.78 +92.31

At 16:35 d O 6215.47 H 6317.02 L 6215.44 Prev 6215.47

UKX Index 95) Templates 96) Actions 97) Expiry Option Monitor: Custom Option Monit

FTSE 100 INDEX 6307.78 / Hi 6317.02 Lo 6215.44 Volm 649718 HV 19.56 91) News (CN)

Calc Mode Center 6307.78 Strikes 5 Exch NYSE Liffe Lon 92) Earnings Calendar(ACDR)

295) Center Strike 296) Calls/Puts 297) Calls 298) Puts 299) Term Structure

Calls Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt	Strike	Puts Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
19 Jul 13 (18d); CSize 10; IDiv 7.47; R .49; IFwd 6297.62								5	19 Jul 13 (18d); CSize 10; IDiv 7.47; R .49; IFwd 6297.62							
UKX 7 C6250	123.5	125.5	124.0s	17.63	.59	84	1849	6250	UKX 7 P6250	74.0	76.0	75.0s	17.49	-.40	231	3240
UKX 7 C6275	108.0	110.0	108.5s	17.27	.56	54	285	6275	UKX 7 P6275	84.0	86.0	84.5s	17.26	-.44	48	157
UKX 7 C6300	94.0	96.0	95.0s	17.00	.52	1951	7870	6300	UKX 7 P6300	94.5	97.0	96.0s	16.92	-.48	1108	5532
UKX 7 C6325	80.5	83.0	82.0s	16.86	.47	128	5713	6325	UKX 7 P6325	106.0	109.0	108.0s	16.73	-.52		5292
UKX 7 C6350	69.0	71.0	70.0s	16.58	.43	49	14167	6350	UKX 7 P6350	119.0	122.5	121.0s	16.53	-.57	3	12821
16 Aug 13 (46d); CSize 10; IDiv 11.56; R .50; IFwd 6265.06								5	16 Aug 13 (46d); CSize 10; IDiv 11.56; R .50; IFwd 6265.06							
UKX 8 C6250	156.5	160.0	157.5s	17.00	.53	10	382	6250	UKX 8 P6250	140.5	143.5	142.0s	17.00	-.46	182	501
UKX 8 C6275	142.0	145.5	144.0s	16.76	.50			6275	UKX 8 P6275	151.5	154.5	153.0s	16.81	-.48		
UKX 8 C6300	129.0	132.0	130.5s	16.52	.48	383	5265	6300	UKX 8 P6300	162.5	166.0	165.0s	16.58	-.51	1	3440
» UKX 8 C6325	116.0	119.5	117.5s	16.33	.45		40	6325	UKX 8 P6325	175.5	179.0	177.0s	16.45	-.54		
UKX 8 C6350	104.5	107.5	106.0s	16.14	.42	336	1693	6350	UKX 8 P6350	188.0	191.5	190.5s	16.25	-.56	31	636
20 Sep 13 (81d); CSize 10; IDiv 10.87; R .51; IFwd 6250.22								5	20 Sep 13 (81d); CSize 10; IDiv 10.87; R .51; IFwd 6250.22							
UKX 9 C6200	224.0	228.0	225.5s	17.33	.55	406	8604	6200	UKX 9 P6200	173.0	176.0	174.5s	17.27	-.43		13635
UKX 9 C6250	194.0	198.0	196.0s	16.93	.51		784	6250	UKX 9 P6250	193.5	196.5	195.0s	16.90	-.47	172	303
UKX 9 C6300	167.5	170.0	169.0s	16.49	.47	687	15371	6300	UKX 9 P6300	216.0	219.0	217.5s	16.57	-.51	3	18270
UKX 9 C6350	142.5	144.5	143.5s	16.11	.43	32	3233	6350	UKX 9 P6350	240.0	244.0	242.5s	16.26	-.55		958
UKX 9 C6400	120.0	122.0	121.0s	15.81	.39	1321	22418	6400	UKX 9 P6400	267.5	272.0	270.0s	16.01	-.59	283	11653
18 Oct 13 (109d); CSize 10; IDiv 10.05; R .53; IFwd 6249.22								5	18 Oct 13 (109d); CSize 10; IDiv 10.05; R .53; IFwd 6249.22							
UKX 10 C6200	252.5	258.0	254.5s	17.34	.54		6	6200	UKX 10 P6200	203.5	208.5	205.0s	17.28	-.43	7	6
UKX 10 C6250	223.5	228.5	225.5s	16.95	.51		25	6250	UKX 10 P6250	224.0	229.0	226.0s	17.04	-.46		
UKX 10 C6300	197.0	201.0	198.0s	16.60	.47		19	6300	UKX 10 P6300	246.5	251.5	248.5s	16.70	-.50		
UKX 10 C6350	171.5	175.5	172.5s	16.28	.44		12	6350	UKX 10 P6350	270.5	276.0	273.0s	16.41	-.53		

93) Default color legend Zoom – 85%

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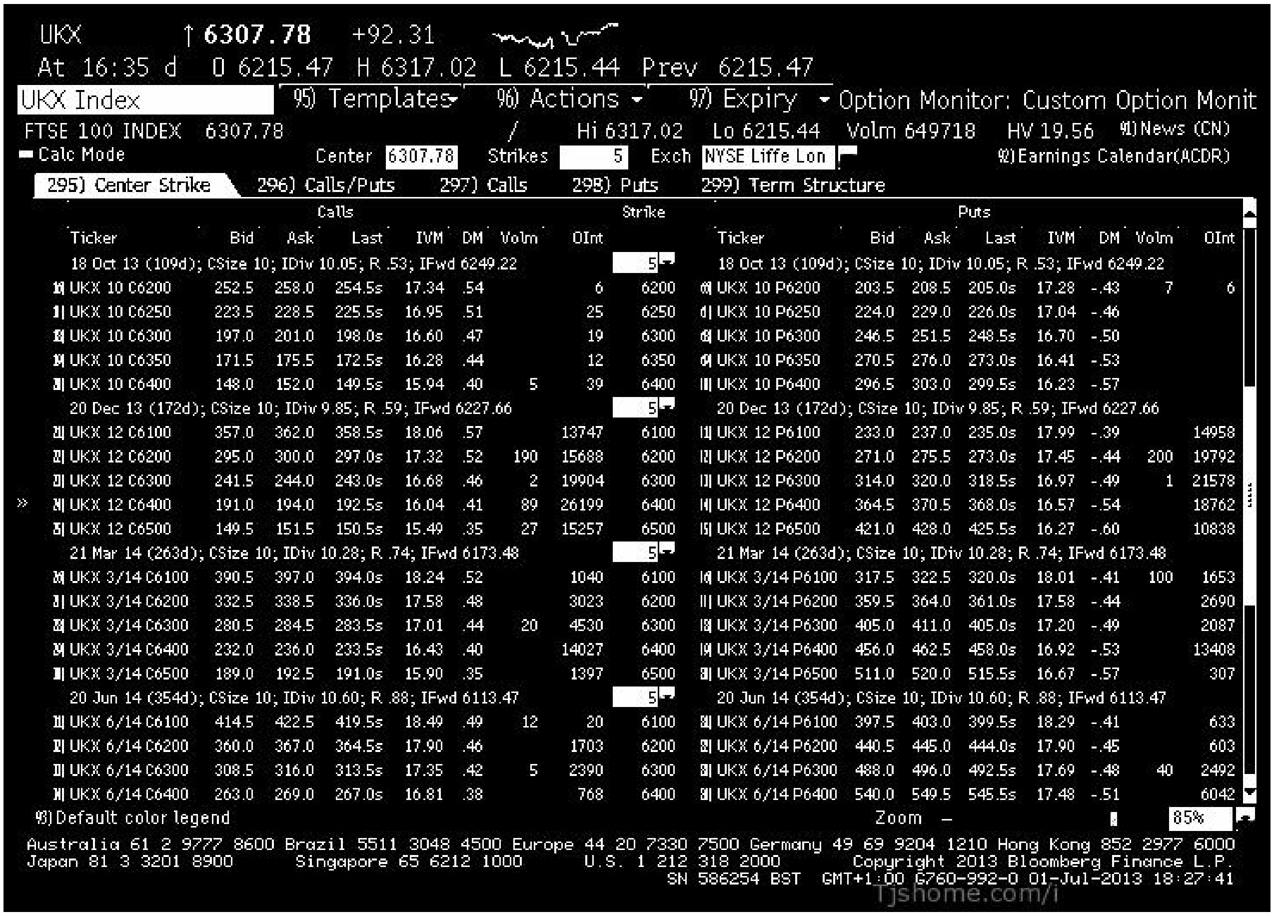
Figure C-4: Options on FTSE 1/3



UKX ↑ 6307.78 +92.31

At 16:35 d O 6215.47 H 6317.02 L 6215.44 Prev 6215.47

UKX Index 95) Templates 96) Actions 97) Expiry Option Monitor: Custom Option Monit

FTSE 100 INDEX 6307.78 / Hi 6317.02 Lo 6215.44 Volm 649718 HV 19.56 91) News (CN)

Calc Mode Center 6307.78 Strikes 5 Exch NYSE Liffe Lon 92) Earnings Calendar(ACDR)

295) Center Strike 296) Calls/Puts 297) Calls 298) Puts 299) Term Structure

Calls Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt	Strike	Puts Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
18 Oct 13 (109d); CSize 10; IDiv 10.05; R .53; IFwd 6249.22								5	18 Oct 13 (109d); CSize 10; IDiv 10.05; R .53; IFwd 6249.22							
UKX 10 C6200	252.5	258.0	254.5s	17.34	.54		6	6200	UKX 10 P6200	203.5	208.5	205.0s	17.28	-.43	7	6
UKX 10 C6250	223.5	228.5	225.5s	16.95	.51		25	6250	UKX 10 P6250	224.0	229.0	226.0s	17.04	-.46		
UKX 10 C6300	197.0	201.0	198.0s	16.60	.47		19	6300	UKX 10 P6300	246.5	251.5	248.5s	16.70	-.50		
UKX 10 C6350	171.5	175.5	172.5s	16.28	.44		12	6350	UKX 10 P6350	270.5	276.0	273.0s	16.41	-.53		
UKX 10 C6400	148.0	152.0	149.5s	15.94	.40	5	39	6400	UKX 10 P6400	296.5	303.0	299.5s	16.23	-.57		
20 Dec 13 (172d); CSize 10; IDiv 9.85; R .59; IFwd 6227.66								5	20 Dec 13 (172d); CSize 10; IDiv 9.85; R .59; IFwd 6227.66							
UKX 12 C6100	357.0	362.0	358.5s	18.06	.57		13747	6100	UKX 12 P6100	233.0	237.0	235.0s	17.99	-.39		14958
UKX 12 C6200	295.0	300.0	297.0s	17.32	.52	190	15688	6200	UKX 12 P6200	271.0	275.5	273.0s	17.45	-.44	200	19792
UKX 12 C6300	241.5	244.0	243.0s	16.68	.46	2	19904	6300	UKX 12 P6300	314.0	320.0	318.5s	16.97	-.49	1	21578
» UKX 12 C6400	191.0	194.0	192.5s	16.04	.41	89	26199	6400	UKX 12 P6400	364.5	370.5	368.0s	16.57	-.54		18762
UKX 12 C6500	149.5	151.5	150.5s	15.49	.35	27	15257	6500	UKX 12 P6500	421.0	428.0	425.5s	16.27	-.60		10838
21 Mar 14 (263d); CSize 10; IDiv 10.28; R .74; IFwd 6173.48								5	21 Mar 14 (263d); CSize 10; IDiv 10.28; R .74; IFwd 6173.48							
UKX 3/14 C6100	390.5	397.0	394.0s	18.24	.52		1040	6100	UKX 3/14 P6100	317.5	322.5	320.0s	18.01	-.41	100	1653
UKX 3/14 C6200	332.5	338.5	336.0s	17.58	.48		3023	6200	UKX 3/14 P6200	359.5	364.0	361.0s	17.58	-.44		2690
UKX 3/14 C6300	280.5	284.5	283.5s	17.01	.44	20	4530	6300	UKX 3/14 P6300	405.0	411.0	405.0s	17.20	-.49		2087
UKX 3/14 C6400	232.0	236.0	233.5s	16.43	.40		14027	6400	UKX 3/14 P6400	456.0	462.5	458.0s	16.92	-.53		13408
UKX 3/14 C6500	189.0	192.5	191.0s	15.90	.35		1397	6500	UKX 3/14 P6500	511.0	520.0	515.5s	16.67	-.57		307
20 Jun 14 (354d); CSize 10; IDiv 10.60; R .88; IFwd 6113.47								5	20 Jun 14 (354d); CSize 10; IDiv 10.60; R .88; IFwd 6113.47							
UKX 6/14 C6100	414.5	422.5	419.5s	18.49	.49	12	20	6100	UKX 6/14 P6100	397.5	403.0	399.5s	18.29	-.41		633
UKX 6/14 C6200	360.0	367.0	364.5s	17.90	.46		1703	6200	UKX 6/14 P6200	440.5	445.0	444.0s	17.90	-.45		603
UKX 6/14 C6300	308.5	316.0	313.5s	17.35	.42	5	2390	6300	UKX 6/14 P6300	488.0	496.0	492.5s	17.69	-.48	40	2492
UKX 6/14 C6400	263.0	269.0	267.0s	16.81	.38		768	6400	UKX 6/14 P6400	540.0	549.5	545.5s	17.48	-.51		6042

93) Default color legend Zoom – 85%

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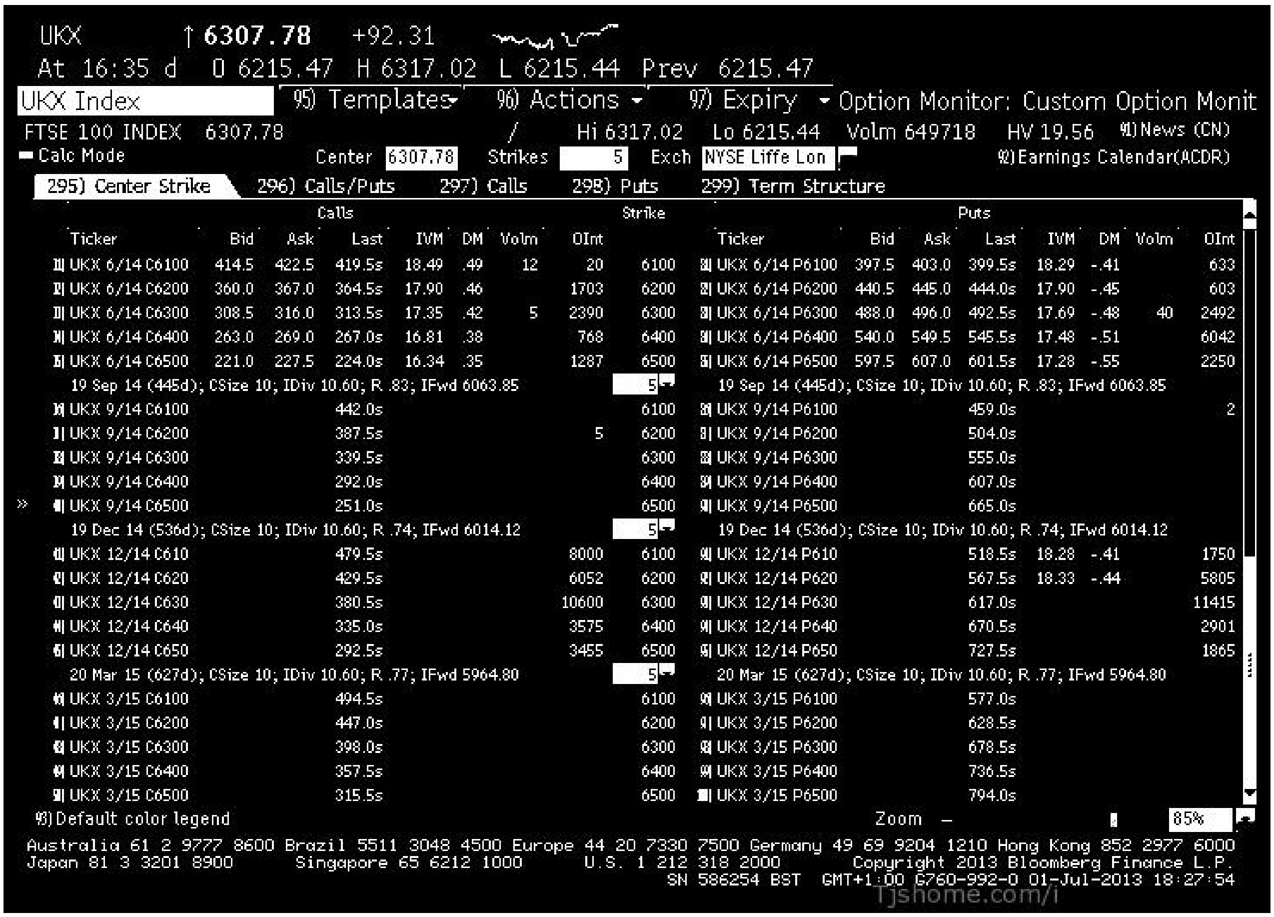
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2013 Bloomberg Finance L.P.
SN 586254 BST GMT+1:00 G760-992-0 01-Jul-2013 18:27:41

Figure C-5: Options on FTSE 2/3



UKX ↑ 6307.78 +92.31
At 16:35 d O 6215.47 H 6317.02 L 6215.44 Prev 6215.47
UKX Index 95) Templates 96) Actions 97) Expiry Option Monitor: Custom Option Monit
FTSE 100 INDEX 6307.78 / Hi 6317.02 Lo 6215.44 Volm 649718 HV 19.56 91) News (CN)
Calc Mode Center 6307.78 Strikes 5 Exch NYSE Liffe Lon 92) Earnings Calendar(ACDR)
295) Center Strike 296) Calls/Puts 297) Calls 298) Puts 299) Term Structure

Calls								Strike	Puts							
Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt		Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
UKX 6/14 C6100	414.5	422.5	419.5s	18.49	.49	12	20	6100	UKX 6/14 P6100	397.5	403.0	399.5s	18.29	-.41		633
UKX 6/14 C6200	360.0	367.0	364.5s	17.90	.46		1703	6200	UKX 6/14 P6200	440.5	445.0	444.0s	17.90	-.45		603
UKX 6/14 C6300	308.5	316.0	313.5s	17.35	.42	5	2390	6300	UKX 6/14 P6300	488.0	496.0	492.5s	17.69	-.48	40	2492
UKX 6/14 C6400	263.0	269.0	267.0s	16.81	.38		768	6400	UKX 6/14 P6400	540.0	549.5	545.5s	17.48	-.51		6042
UKX 6/14 C6500	221.0	227.5	224.0s	16.34	.35		1287	6500	UKX 6/14 P6500	597.5	607.0	601.5s	17.28	-.55		2250
19 Sep 14 (445d); CSize 10; IDiv 10.60; R .83; IFwd 6063.85								5	19 Sep 14 (445d); CSize 10; IDiv 10.60; R .83; IFwd 6063.85							
UKX 9/14 C6100			442.0s					6100	UKX 9/14 P6100			459.0s				2
UKX 9/14 C6200			387.5s				5	6200	UKX 9/14 P6200			504.0s				
UKX 9/14 C6300			339.5s					6300	UKX 9/14 P6300			555.0s				
UKX 9/14 C6400			292.0s					6400	UKX 9/14 P6400			607.0s				
» UKX 9/14 C6500			251.0s					6500	UKX 9/14 P6500			665.0s				
19 Dec 14 (536d); CSize 10; IDiv 10.60; R .74; IFwd 6014.12								5	19 Dec 14 (536d); CSize 10; IDiv 10.60; R .74; IFwd 6014.12							
UKX 12/14 C610			479.5s				8000	6100	UKX 12/14 P610			518.5s	18.28	-.41		1750
UKX 12/14 C620			429.5s				6052	6200	UKX 12/14 P620			567.5s	18.33	-.44		5805
UKX 12/14 C630			380.5s				10600	6300	UKX 12/14 P630			617.0s				11415
UKX 12/14 C640			335.0s				3575	6400	UKX 12/14 P640			670.5s				2901
UKX 12/14 C650			292.5s				3455	6500	UKX 12/14 P650			727.5s				1865
20 Mar 15 (627d); CSize 10; IDiv 10.60; R .77; IFwd 5964.80								5	20 Mar 15 (627d); CSize 10; IDiv 10.60; R .77; IFwd 5964.80							
UKX 3/15 C6100			494.5s					6100	UKX 3/15 P6100			577.0s				
UKX 3/15 C6200			447.0s					6200	UKX 3/15 P6200			628.5s				
UKX 3/15 C6300			398.0s					6300	UKX 3/15 P6300			678.5s				
UKX 3/15 C6400			357.5s					6400	UKX 3/15 P6400			736.5s				
UKX 3/15 C6500			315.5s					6500	UKX 3/15 P6500			794.0s				

93) Default color legend Zoom – 85%
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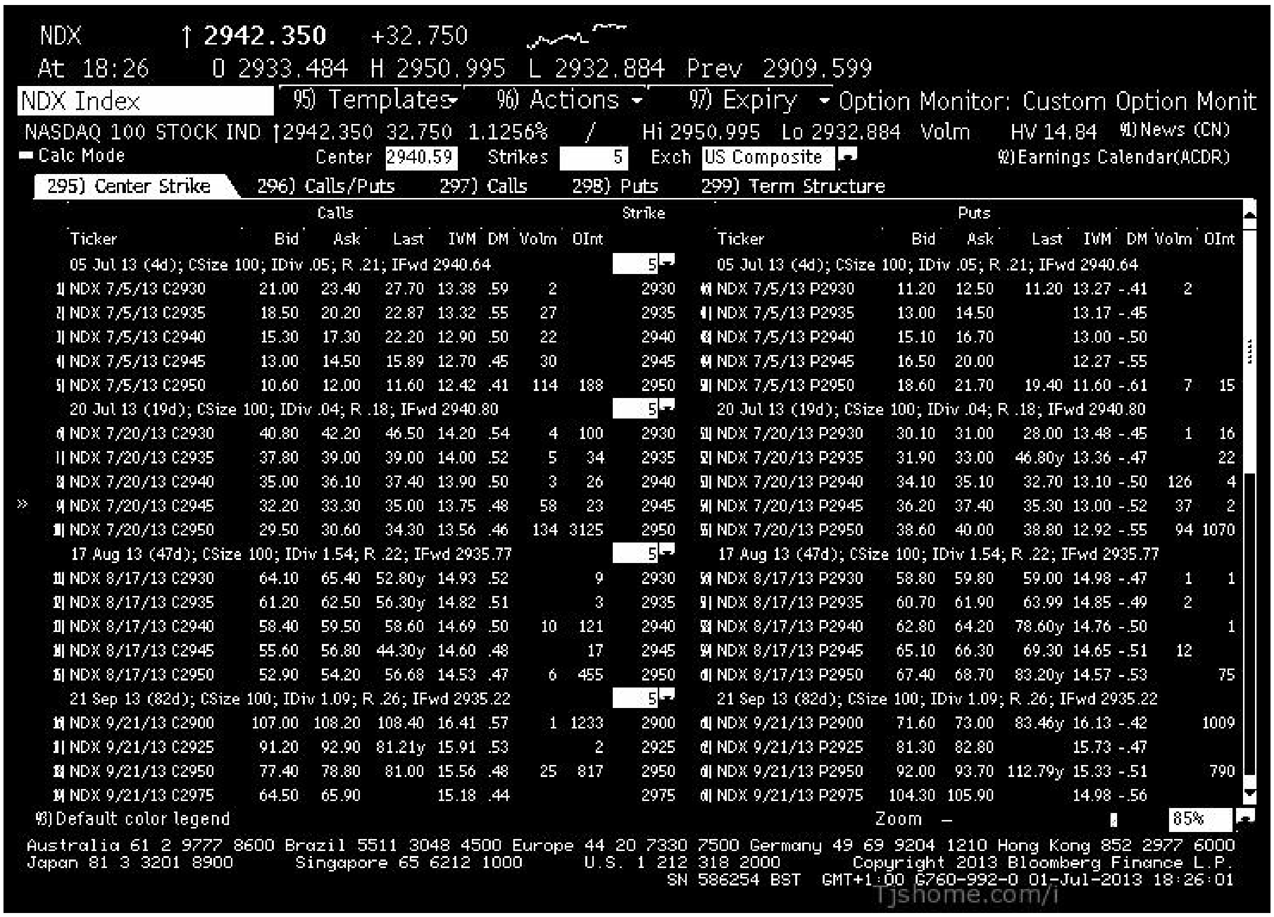
Figure C-6: Options on FTSE 3/3



NDX ↑ 2942.350 +32.750
At 18:26 O 2933.484 H 2950.995 L 2932.884 Prev 2909.599
NDX Index 95) Templates 96) Actions 97) Expiry Option Monitor: Custom Option Monit
NASDAQ 100 STOCK IND ↑2942.350 32.750 1.1256% / Hi 2950.995 Lo 2932.884 Volm HV 14.84 91) News (CN)
Calc Mode Center 2940.59 Strikes 5 Exch US Composite 92) Earnings Calendar(ACDR)
295) Center Strike 296) Calls/Puts 297) Calls 298) Puts 299) Term Structure

Calls Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt	Strike	Puts Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
05 Jul 13 (4d); CSize 100; IDiv .05; R .21; IFwd 2940.64								5	05 Jul 13 (4d); CSize 100; IDiv .05; R .21; IFwd 2940.64							
NDX 7/5/13 C2930	21.00	23.40	27.70	13.38	.59	2		2930	NDX 7/5/13 P2930	11.20	12.50	11.20	13.27	-.41	2	
NDX 7/5/13 C2935	18.50	20.20	22.87	13.32	.55	27		2935	NDX 7/5/13 P2935	13.00	14.50		13.17	-.45		
NDX 7/5/13 C2940	15.30	17.30	22.20	12.90	.50	22		2940	NDX 7/5/13 P2940	15.10	16.70		13.00	-.50		
NDX 7/5/13 C2945	13.00	14.50	15.89	12.70	.45	30		2945	NDX 7/5/13 P2945	16.50	20.00		12.27	-.55		
NDX 7/5/13 C2950	10.60	12.00	11.60	12.42	.41	114	188	2950	NDX 7/5/13 P2950	18.60	21.70	19.40	11.60	-.61	7	15
20 Jul 13 (19d); CSize 100; IDiv .04; R .18; IFwd 2940.80								5	20 Jul 13 (19d); CSize 100; IDiv .04; R .18; IFwd 2940.80							
NDX 7/20/13 C2930	40.80	42.20	46.50	14.20	.54	4	100	2930	NDX 7/20/13 P2930	30.10	31.00	28.00	13.48	-.45	1	16
NDX 7/20/13 C2935	37.80	39.00	39.00	14.00	.52	5	34	2935	NDX 7/20/13 P2935	31.90	33.00	46.80y	13.36	-.47		22
NDX 7/20/13 C2940	35.00	36.10	37.40	13.90	.50	3	26	2940	NDX 7/20/13 P2940	34.10	35.10	32.70	13.10	-.50	126	4
NDX 7/20/13 C2945	32.20	33.30	35.00	13.75	.48	58	23	2945	NDX 7/20/13 P2945	36.20	37.40	35.30	13.00	-.52	37	2
NDX 7/20/13 C2950	29.50	30.60	34.30	13.56	.46	134	3125	2950	NDX 7/20/13 P2950	38.60	40.00	38.80	12.92	-.55	94	1070
17 Aug 13 (47d); CSize 100; IDiv 1.54; R .22; IFwd 2935.77								5	17 Aug 13 (47d); CSize 100; IDiv 1.54; R .22; IFwd 2935.77							
NDX 8/17/13 C2930	64.10	65.40	52.80y	14.93	.52		9	2930	NDX 8/17/13 P2930	58.80	59.80	59.00	14.98	-.47	1	1
NDX 8/17/13 C2935	61.20	62.50	56.30y	14.82	.51		3	2935	NDX 8/17/13 P2935	60.70	61.90	63.99	14.85	-.49	2	
NDX 8/17/13 C2940	58.40	59.50	58.60	14.69	.50	10	121	2940	NDX 8/17/13 P2940	62.80	64.20	78.60y	14.76	-.50		1
NDX 8/17/13 C2945	55.60	56.80	44.30y	14.60	.48		17	2945	NDX 8/17/13 P2945	65.10	66.30	69.30	14.65	-.51	12	
NDX 8/17/13 C2950	52.90	54.20	56.68	14.53	.47	6	455	2950	NDX 8/17/13 P2950	67.40	68.70	83.20y	14.57	-.53		75
21 Sep 13 (82d); CSize 100; IDiv 1.09; R .26; IFwd 2935.22								5	21 Sep 13 (82d); CSize 100; IDiv 1.09; R .26; IFwd 2935.22							
NDX 9/21/13 C2900	107.00	108.20	108.40	16.41	.57	1	1233	2900	NDX 9/21/13 P2900	71.60	73.00	83.46y	16.13	-.42		1009
NDX 9/21/13 C2925	91.20	92.90	81.21y	15.91	.53		2	2925	NDX 9/21/13 P2925	81.30	82.80		15.73	-.47		
NDX 9/21/13 C2950	77.40	78.80	81.00	15.56	.48	25	817	2950	NDX 9/21/13 P2950	92.00	93.70	112.79y	15.33	-.51		790
NDX 9/21/13 C2975	64.50	65.90		15.18	.44			2975	NDX 9/21/13 P2975	104.30	105.90		14.98	-.56		

93) Default color legend Zoom 85%

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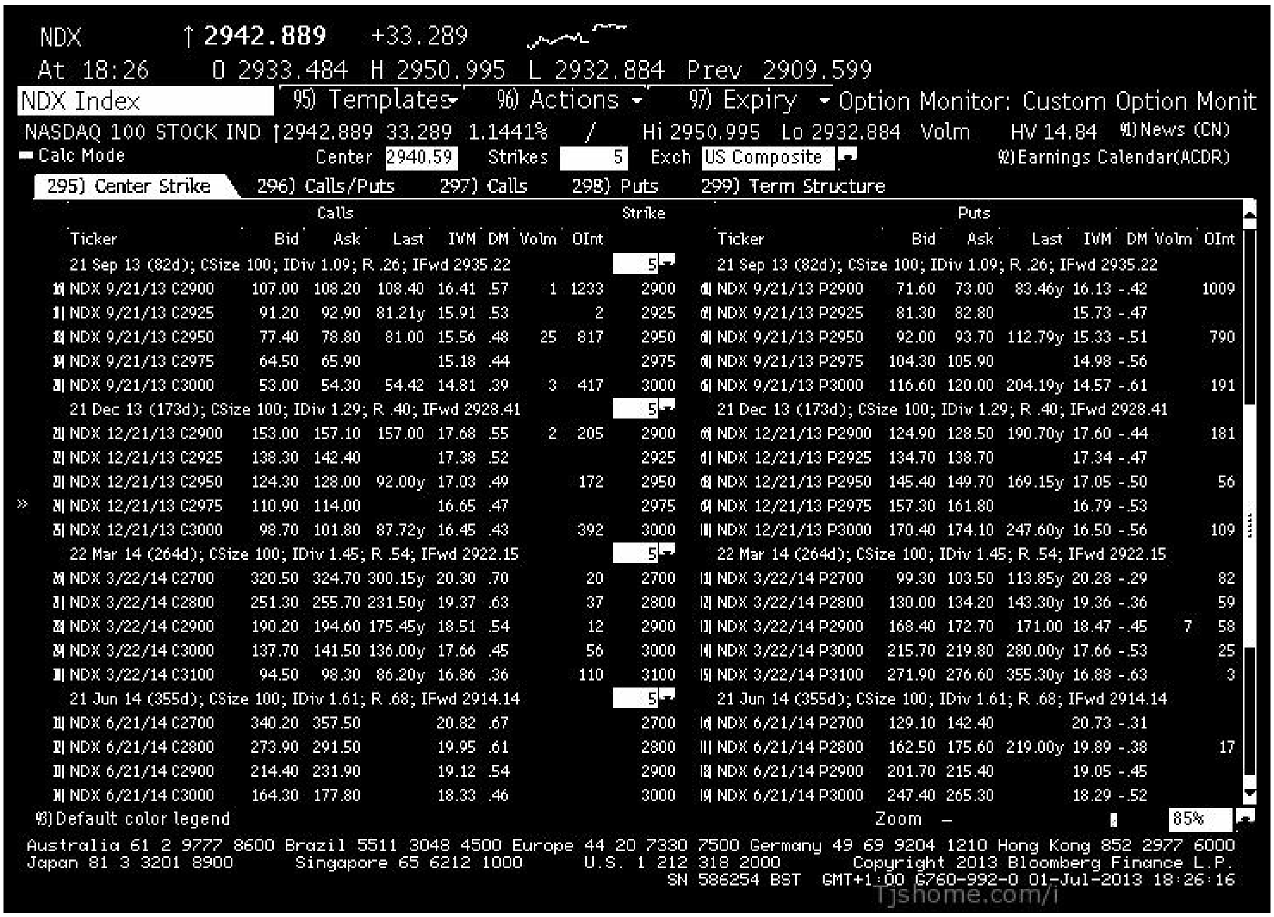
Figure C-7: Options on Nasdaq 1/3

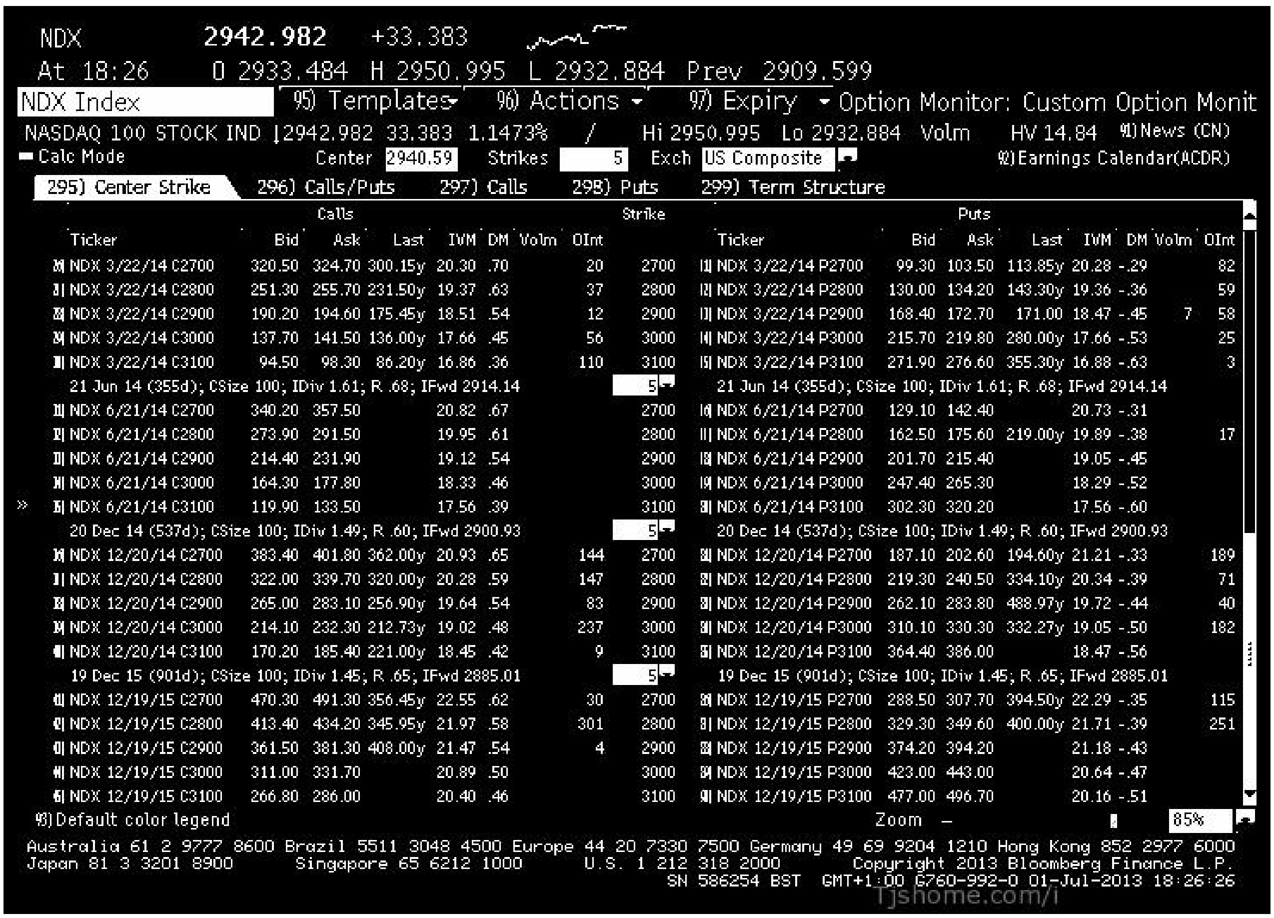


NDX ↑ 2942.889 +33.289
At 18:26 O 2933.484 H 2950.995 L 2932.884 Prev 2909.599
NDX Index 95) Templates 96) Actions 97) Expiry Option Monitor: Custom Option Monit
NASDAQ 100 STOCK IND ↑2942.889 33.289 1.1441% / Hi 2950.995 Lo 2932.884 Volm HV 14.84 91) News (CN)
Calc Mode Center 2940.59 Strikes 5 Exch US Composite 92) Earnings Calendar(ACDR)
295) Center Strike 296) Calls/Puts 297) Calls 298) Puts 299) Term Structure

Calls								Strike	Puts							
Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt		Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
21 Sep 13 (82d); CSize 100; IDiv 1.09; R .26; IFwd 2935.22								5	21 Sep 13 (82d); CSize 100; IDiv 1.09; R .26; IFwd 2935.22							
NDX 9/21/13 C2900	107.00	108.20	108.40	16.41	.57	1	1233	2900	NDX 9/21/13 P2900	71.60	73.00	83.46y	16.13	-.42		1009
NDX 9/21/13 C2925	91.20	92.90	81.21y	15.91	.53		2	2925	NDX 9/21/13 P2925	81.30	82.80		15.73	-.47		
NDX 9/21/13 C2950	77.40	78.80	81.00	15.56	.48	25	817	2950	NDX 9/21/13 P2950	92.00	93.70	112.79y	15.33	-.51		790
NDX 9/21/13 C2975	64.50	65.90		15.18	.44			2975	NDX 9/21/13 P2975	104.30	105.90		14.98	-.56		
NDX 9/21/13 C3000	53.00	54.30	54.42	14.81	.39	3	417	3000	NDX 9/21/13 P3000	116.60	120.00	204.19y	14.57	-.61		191
21 Dec 13 (173d); CSize 100; IDiv 1.29; R .40; IFwd 2928.41								5	21 Dec 13 (173d); CSize 100; IDiv 1.29; R .40; IFwd 2928.41							
NDX 12/21/13 C2900	153.00	157.10	157.00	17.68	.55	2	205	2900	NDX 12/21/13 P2900	124.90	128.50	190.70y	17.60	-.44		181
NDX 12/21/13 C2925	138.30	142.40		17.38	.52			2925	NDX 12/21/13 P2925	134.70	138.70		17.34	-.47		
NDX 12/21/13 C2950	124.30	128.00	92.00y	17.03	.49		172	2950	NDX 12/21/13 P2950	145.40	149.70	169.15y	17.05	-.50		56
» NDX 12/21/13 C2975	110.90	114.00		16.65	.47			2975	NDX 12/21/13 P2975	157.30	161.80		16.79	-.53		
NDX 12/21/13 C3000	98.70	101.80	87.72y	16.45	.43		392	3000	NDX 12/21/13 P3000	170.40	174.10	247.60y	16.50	-.56		109
22 Mar 14 (264d); CSize 100; IDiv 1.45; R .54; IFwd 2922.15								5	22 Mar 14 (264d); CSize 100; IDiv 1.45; R .54; IFwd 2922.15							
NDX 3/22/14 C2700	320.50	324.70	300.15y	20.30	.70		20	2700	NDX 3/22/14 P2700	99.30	103.50	113.85y	20.28	-.29		82
NDX 3/22/14 C2800	251.30	255.70	231.50y	19.37	.63		37	2800	NDX 3/22/14 P2800	130.00	134.20	143.30y	19.36	-.36		59
NDX 3/22/14 C2900	190.20	194.60	175.45y	18.51	.54		12	2900	NDX 3/22/14 P2900	168.40	172.70	171.00	18.47	-.45	7	58
NDX 3/22/14 C3000	137.70	141.50	136.00y	17.66	.45		56	3000	NDX 3/22/14 P3000	215.70	219.80	280.00y	17.66	-.53		25
NDX 3/22/14 C3100	94.50	98.30	86.20y	16.86	.36		110	3100	NDX 3/22/14 P3100	271.90	276.60	355.30y	16.88	-.63		3
21 Jun 14 (355d); CSize 100; IDiv 1.61; R .68; IFwd 2914.14								5	21 Jun 14 (355d); CSize 100; IDiv 1.61; R .68; IFwd 2914.14							
NDX 6/21/14 C2700	340.20	357.50		20.82	.67			2700	NDX 6/21/14 P2700	129.10	142.40		20.73	-.31		
NDX 6/21/14 C2800	273.90	291.50		19.95	.61			2800	NDX 6/21/14 P2800	162.50	175.60	219.00y	19.89	-.38		17
NDX 6/21/14 C2900	214.40	231.90		19.12	.54			2900	NDX 6/21/14 P2900	201.70	215.40		19.05	-.45		
NDX 6/21/14 C3000	164.30	177.80		18.33	.46			3000	NDX 6/21/14 P3000	247.40	265.30		18.29	-.52		

93) Default color legend Zoom – 85%
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2013 Bloomberg Finance L.P.
SN 586254 BST GMT+1:00 G760-992-0 01-Jul-2013 18:26:16
Tjshome.com/i

Figure C-8: Options on Nasdaq 2/3



NDX 2942.982 +33.383
At 18:26 O 2933.484 H 2950.995 L 2932.884 Prev 2909.599
NDX Index 95) Templates 96) Actions 97) Expiry Option Monitor: Custom Option Monit
NASDAQ 100 STOCK IND 2942.982 33.383 1.1473% / Hi 2950.995 Lo 2932.884 Volm HV 14.84 91) News (CN)
Calc Mode Center 2940.59 Strikes 5 Exch US Composite 92) Earnings Calendar(ACDR)
295) Center Strike 296) Calls/Puts 297) Calls 298) Puts 299) Term Structure

Calls								Strike	Puts							
Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt		Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
NDX 3/22/14 C2700	320.50	324.70	300.15y	20.30	.70		20	2700	NDX 3/22/14 P2700	99.30	103.50	113.85y	20.28	-.29		82
NDX 3/22/14 C2800	251.30	255.70	231.50y	19.37	.63		37	2800	NDX 3/22/14 P2800	130.00	134.20	143.30y	19.36	-.36		59
NDX 3/22/14 C2900	190.20	194.60	175.45y	18.51	.54		12	2900	NDX 3/22/14 P2900	168.40	172.70	171.00	18.47	-.45	7	58
NDX 3/22/14 C3000	137.70	141.50	136.00y	17.66	.45		56	3000	NDX 3/22/14 P3000	215.70	219.80	280.00y	17.66	-.53		25
NDX 3/22/14 C3100	94.50	98.30	86.20y	16.86	.36		110	3100	NDX 3/22/14 P3100	271.90	276.60	355.30y	16.88	-.63		3
21 Jun 14 (355d); CSize 100; IDiv 1.61; R .68; IFwd 2914.14								5	21 Jun 14 (355d); CSize 100; IDiv 1.61; R .68; IFwd 2914.14							
NDX 6/21/14 C2700	340.20	357.50		20.82	.67			2700	NDX 6/21/14 P2700	129.10	142.40		20.73	-.31		
NDX 6/21/14 C2800	273.90	291.50		19.95	.61			2800	NDX 6/21/14 P2800	162.50	175.60	219.00y	19.89	-.38		17
NDX 6/21/14 C2900	214.40	231.90		19.12	.54			2900	NDX 6/21/14 P2900	201.70	215.40		19.05	-.45		
NDX 6/21/14 C3000	164.30	177.80		18.33	.46			3000	NDX 6/21/14 P3000	247.40	265.30		18.29	-.52		
» NDX 6/21/14 C3100	119.90	133.50		17.56	.39			3100	NDX 6/21/14 P3100	302.30	320.20		17.56	-.60		
20 Dec 14 (537d); CSize 100; IDiv 1.49; R .60; IFwd 2900.93								5	20 Dec 14 (537d); CSize 100; IDiv 1.49; R .60; IFwd 2900.93							
NDX 12/20/14 C2700	383.40	401.80	362.00y	20.93	.65		144	2700	NDX 12/20/14 P2700	187.10	202.60	194.60y	21.21	-.33		189
NDX 12/20/14 C2800	322.00	339.70	320.00y	20.28	.59		147	2800	NDX 12/20/14 P2800	219.30	240.50	334.10y	20.34	-.39		71
NDX 12/20/14 C2900	265.00	283.10	256.90y	19.64	.54		83	2900	NDX 12/20/14 P2900	262.10	283.80	488.97y	19.72	-.44		40
NDX 12/20/14 C3000	214.10	232.30	212.73y	19.02	.48		237	3000	NDX 12/20/14 P3000	310.10	330.30	332.27y	19.05	-.50		182
NDX 12/20/14 C3100	170.20	185.40	221.00y	18.45	.42		9	3100	NDX 12/20/14 P3100	364.40	386.00		18.47	-.56		
19 Dec 15 (901d); CSize 100; IDiv 1.45; R .65; IFwd 2885.01								5	19 Dec 15 (901d); CSize 100; IDiv 1.45; R .65; IFwd 2885.01							
NDX 12/19/15 C2700	470.30	491.30	356.45y	22.55	.62		30	2700	NDX 12/19/15 P2700	288.50	307.70	394.50y	22.29	-.35		115
NDX 12/19/15 C2800	413.40	434.20	345.95y	21.97	.58		301	2800	NDX 12/19/15 P2800	329.30	349.60	400.00y	21.71	-.39		251
NDX 12/19/15 C2900	361.50	381.30	408.00y	21.47	.54		4	2900	NDX 12/19/15 P2900	374.20	394.20		21.18	-.43		
NDX 12/19/15 C3000	311.00	331.70		20.89	.50			3000	NDX 12/19/15 P3000	423.00	443.00		20.64	-.47		
NDX 12/19/15 C3100	266.80	286.00		20.40	.46			3100	NDX 12/19/15 P3100	477.00	496.70		20.16	-.51		

93) Default color legend Zoom – 85%
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Figure C-9: Options on Nasdaq 3/3

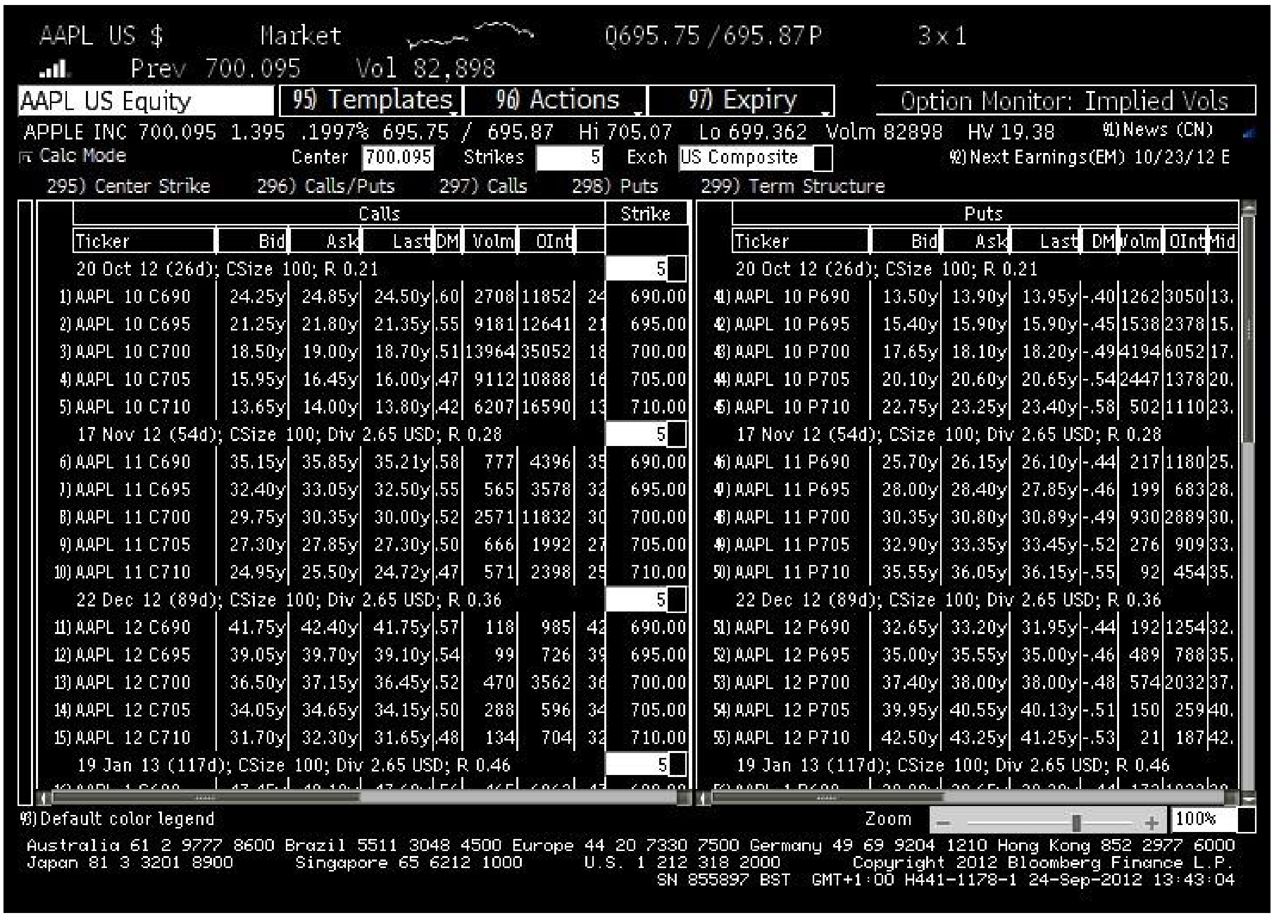

Figure C-10: Options on Apple's on 23/10/2012

Appendix D

Matlab® Code

The Merton Model

Valuation Function

```
function c = MJD(type,So,X,T,delta,nu,lambda,vola,r,N) %% r->r-q
K = exp(nu + 0.5*delta^2) - 1;
c = 0;
for n = 0 : N
    sigma_n = sqrt(vola^2 + n*delta^2/T);
    r_n = r - lambda*K + n*log(1+K)/T;
    f_n = BS(type,So,X,r_n,0,sigma_n,T);
    c = c + exp(-lambda*(1+K)*T) * (lambda*(1+K)*T)^n * f_n/factorial(n);
end
end
```

The Kou Model

Hh Functions

```
function out = Hh(n,x)
if n==-1
    out=sqrt(2*pi)*normpdf(x,0,1);
end
if n==0
    out=sqrt(2*pi)*normcdf(-x,0,1);
end
if n>0
    v(1)=sqrt(2*pi)*normpdf(x,0,1);
    v(2)=sqrt(2*pi)*normcdf(-x,0,1);
    for i=3:n+2
        v(i)=(1/(i-2))*(v(i-2)-x*v(i-1));
    end
    out=v(n+2);
end
end
```

P Functions

```
function out = P(n,i,p,q,eta1,eta2)
if i==n
    out=p^n;
else
    sum=0;
    for j=1:n-1
        sum = sum + p^j*q^(n-j)*nchoosek(n-i-1,j-1)*(eta1/(eta1+eta2))^(j-1)*(eta2/(eta1+eta2))^(n-j);
    end
    out=sum;
end
end
```

D Functions

```
function out = Q(n,i,p,q,eta1,eta2)
if i==n
    out=q^n;
else
    sum=0;
    for j=1:n-1
        sum = sum + q^j*p^(n-j)*nchoosek(n-i-1,j-1)*(eta2/(eta1+eta2))^(j-1)*(eta1/(eta1+eta2))^(n-j);
    end
    out=sum;
end
end
```

I Functions

```
function out = I(n,c,alpha,beta,delta)
sum=0;
for i=1:n
    sum=sum+(beta/alpha)^(n-i)*Hh(i,beta*c-delta);
end
out=sum;
if beta>0 && alpha~=0
    out=-(exp(alpha*c)/alpha)*sum+(beta/alpha)^(n+1)*(sqrt(2*pi)/beta)*exp(alpha*delta/beta+alpha^2/(2*beta^2))*normcdf(-beta*c+delta+alpha/beta,0,1);
else if beta<0 && alpha<0
        out=-(exp(alpha*c)/alpha)*sum+(beta/alpha)^(n+1)*(sqrt(2*pi)/beta)*exp(alpha*delta/beta+alpha^2/(2*beta^2))*normcdf(beta*c-delta-alpha/beta,0,1);
```

```
        else
            out=0;%'non valid arguments'
        end
end
```

π Functions

```
function out=Pi(n,lambda,T)
out=(exp(-lambda*T)^lambda^n)/(factorial(n));
end
```

Ψ Functions

```
function out = Psi(mu,sigma,lambda,p,q,eta1,eta2,a,T)
sum_old= Pi(0,lambda,T)*normcdf(-(a-mu*T)/(sigma*sqrt(T)),0,1);
n=1;
dif=1;
out=sum_old;
end
```

Valuation Function

```
function out = Kou(type,So,X,r,sigma,lambda,p,eta1,eta2,T) %% r-> r-q
q=1-p;
if eta1<=1 || eta2<=0
    out=0;
else
    zeta=(p*eta1)/(eta1-1)+(q*eta2)/(eta2+1)-1;
    p_tilde=(p/(1+zeta))*(eta1/(eta1-1));
    q_tilde=1-p_tilde;
    eta1_tilde=eta1-1;
```

```
    eta2_tilde=eta2+1;
    lambda_tilde=lambda*(zeta+1);
    call=So*Psi(r+sigma^2/2-lambda*zeta,sigma,lambda_tilde,
        p_tilde,q_tilde,eta1_tilde,eta2_tilde,log(X/So),T) -
            X*exp(-r*T)*Psi(r-sigma^2/2-lambda*zeta,sigma,
        lambda,p,q,eta1,eta2,log(X/So),T);
    if type==1
        out = call;
    else
        put = X*exp(-r*T)-So+call;
        out=put;
    end
end
```

The Variance Gamma Model

Valuation Function

```
function out = VG(type,So,X,r,T,theta,nu,sigma)
CF_VG =@(u) (1./(1-1i.*u.*theta.*nu+sigma^2*nu/2.*u.^2)).^(T/
    nu);
CF_VG_lgST=@(u) exp( 1i.*u.*( log(So)+r*T+T/nu*log(1-theta*nu
    -sigma^2*nu/2) ) ).*CF_VG(u);
integrand1 =@(u) real( exp(-1i.*u.*log(X)).*CF_VG_lgST(u-1i)
    ./(1i.*u.*CF_VG_lgST(-1i)) );
integrand2 =@(u) real( exp(-1i.*u.*log(X)).*CF_VG_lgST(u)./(1
    i.*u) );
integral1 = quadgk(integrand1,0.01,inf,'RelTol',1e-8,'AbsTol'
    ,1e-12); %'MaxIntervalCount',200000
integral2 = quadgk(integrand2,0.01,inf,'RelTol',1e-8,'AbsTol'
    ,1e-12);
```

```
Prob1 = 0.5 + (1/pi)*integral1;
Prob2 = 0.5 + (1/pi)*integral2;
call = So*Prob1 - X*exp(-r*T)*Prob2;
if type==1
     out = call;
else
     out = call-So+exp(-r*T)*X;
end
end
```

Bibliography

[1] M. Abramowitz and I.A. Stegun. *Handbook of Mathematical Functions.* Dover, New York, fifth edition, 1964.

[2] Tiago Alcaria. Avaliação de Opções via transformadas de Fourier. Master of Science thesis - ISCTE-IUL, 2008.

[3] David Applebaum. *Lévy Processes and Stochastic Calculus.* Cambridge University Press, 2004.

[4] L. Bachelier. Théorie de la spéculation. *Annales scientifiques de l'École Normale Supérieure, Sér. 3*, 3:21–86, 1900.

[5] Gurdip Bakshi, Dilip Madan, Rick Green, Steve Heston, Nengjiu Ju, Nikunj Kapadia, Hossein Kazemi, and Inanc Kirgiz. Spanning and derivative-security valuation. *Journal of Financial Economics* 55 205-238, 2000.

[6] D. S. Bates. Jumps and stochastic volatility: exchange rate processes implicit in Deutsche mark options. in *Review of Financial Studies 9:1, 69-107.*, volume = 9, year = 1996. (1):69–107.

[7] Fischer Black and Myron Scholes. The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, 81(3):637–654, 1973.

[8] Peter P. Carr and Dilip B. Madan. Option Valuation Using the Fast Fourier Transform. *Journal of Computational Finance*, 2:61–73, 1999.

[9] Gerald Cheang and Carl Chiarella. A modern view on Merton's jump-diffusion model. Research Paper Series 287, Quantitative Finance Research Centre, University of Technology, Sydney, 2011.

[10] Rama Cont and Peter Tankov. *Financial Modelling with Jump Processes.* Chapman & Hall/CRC Financial Mathematics Series, 2003.

[11] J. Cox. Notes on Option Pricing I: Constant Elasticity of Variance Diffusions, Working Paper. *(reprinted in Journal of Portfolio Management, 1996, December, 15-17)*, 1975.

[12] Hua Fang. Option pricing implications of a stochastic jump rate, Working Paper,University of Virginia, 2000.

[13] Michael C. Fu. Variance Gamma and Monte Carlo. In Advances in Mathematical Finance (eds. Fu, Jarrow, Yen, Elliott), Birkhuser, 21-35, 2007.

[14] Helyette Geman, Nicole E. Karoui, and Jean C. Rochet. Changes of Numeraire, Changes of Probability Measure and Option Pricing. *Journal of Applied Probaility*, 32:443–458, 1995.

[15] Hans U. Gerber and Elias S. W. Shiu. IV Option Pricing by Esscher Transforms,Working Paper. In Transactions of Society of Actuaries Vol. 46. 1994.

[16] Steven L. Heston. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6:327–343, 1993.

[17] S. G. Kou. A Jump-Diffusion Model for Option Pricing. *Manage. Sci.*, 48(8):1086–1101, 2002.

[18] S. G. Kou and Hui Wang. Option pricing under a double exponential jump diffusion model. *Management Science*, 50(9):1178–1192, 2004.

[19] Dilip B. Madan. Purely discontinuous asset price processes. In *Option Pricing, Interest Rates and Risk Management*, pages 105–153. University Press, 1999.

[20] Dilip B. Madan, Peter Carr, and Eric C. Chang. The variance gamma process and option pricing. *European Finance Review*, 2:79–105, 1998.

[21] Dilip B Madan and Eugene Seneta. The variance gamma (v.g.) model for share market returns. *The Journal of Business*, 63(4):511–24, 1990.

[22] Benoit Mandelbrot and Richard L. Hudson. *The Misbehavior of Markets: A Fractal View of Financial Turbulence*. Basic Books, annotated edition, March 2006.

[23] Michael Mastro. *Financial Derivative and Energy Market Valuation: Theory and Implemantation in Matlab*. Wiley, 2013.

[24] Kazuhisa Matsuda. Introduction to Merton jump diffusion model, Working Paper, The City University of New York, 2004.

[25] Robert C. Merton. Option pricing when underlying stock returns are discontinuous. Technical report, 1975.

[26] William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery. *Numerical Recipes 3rd Edition: The Art of Scientific Computing*. Cambridge University Press, New York, NY, USA, 3 edition, 2007.

[27] P. Tankov and E. Voltchkova. Jump-Diffusion Models: A Practitioner's Guide. Technical report, Université Paris VII, 2006.